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Large and Medium Power Synchronous Generators: Topologies and Steady State

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4.1 Introduction

By large powers, we mean here powers above 1 MW per unit, where in general, the rotor magnetic field is produced with electromagnetic excitation. There are a few megawatt (MW) power permanent magnet (PM)-rotor synchronous generators (SGs).

Almost all electric energy generation is performed through SGs with power per unit up to 1500 MVA in thermal power plants and up to 700 MW per unit in hydropower plants. SGs in the MW and tenth of MW range are used in diesel engine power groups for cogeneration and on locomotives and on ships. We will begin with a description of basic configurations, their main components, and principles of operation, and then describe the steady-state operation in detail.

4.2 Construction Elements

The basic parts of an SG are the stator, the rotor, the framing (with cooling system), and the excitation system.

The stator is provided with a magnetic core made of silicon steel sheets (generally 0.55 mm thick) in which uniform slots are stamped. Single, standard, magnetic sheet steel is produced up to 1 m in diameter in the form of a complete circle (Figure 4.1). Large turbogenerators and most hydrogenerators have stator outer diameters well in excess of 1 m (up to 18 m); thus, the cores are made of 6 to 42 segments per circle (Figure 4.2).

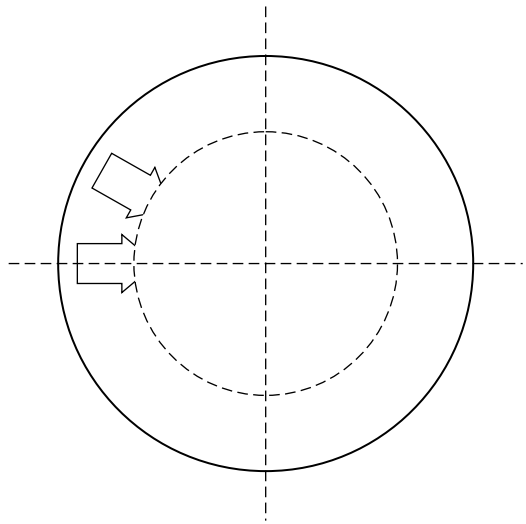


FIGURE 4.1 Single piece stator core.

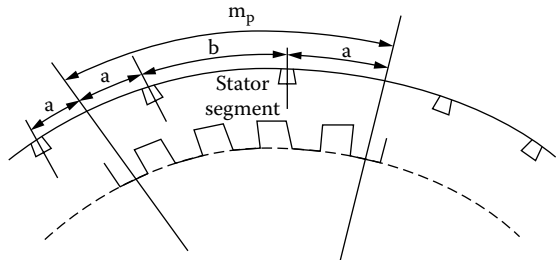


FIGURE 4.2 Divided stator core made of segments.

The stator may also be split radially into two or more sections to allow handling and permit transport with windings in slots. The windings in slots are inserted section by section, and their connection is performed at the power plant site.

When the stator with N_s slots is divided, and the number of slot pitches per segment is m_p , the number of segments m_s is such that

$$N_s = m_s \cdot m_p \quad (4.1)$$

Each segment is attached to the frame through two key-bars or dove-tail wedges that are uniformly distributed along the periphery (Figure 4.2). In two successive layers (laminations), the segments are offset by half a segment. The distance between wedges b is as follows:

$$b = m_p / 2 = 2a \quad (4.2)$$

This distance between wedges allows for offsetting the segments in subsequent layers by half a segment. Also, only one tool for stamping is required, because all segments are identical. To avoid winding damage due to vibration, each segment should start and end in the middle of a tooth and span over an even number of slot pitches.

For the stator divided into S sectors, two types of segments are usually used. One type has m_p slot pitches, and the other has n_p slot pitches, such that

$$\frac{N_s}{S} = Km_p + n_p; n_p < m_p; m_p = 6-13 \quad (4.3)$$

With $n_p = 0$, the first case is obtained, and, in fact, the number of segments per stator sector is an integer. This is not always possible, and thus, two types of segments are required.

The offset of segments in subsequent layers is $m_p/2$ if m_p is even, $(m_p \pm 1)/2$ if m_p is odd, and $m_p/3$ if m_p is divisible by three. In the particular case that $n_p = m_p/2$, we may cut the main segment in two to obtain the second one, which again would require only one stamping tool. For more details, see Reference [1].

The slots of large and medium power SGs are rectangular and open (Figure 4.3a).

The double-layer winding, usually made of magnetic wires with rectangular cross-section, is “kept” inside the open slot by a wedge made of insulator material or from a magnetic material with a low equivalent tangential permeability that is μ_r times larger than that of air. The magnetic wedge may be made of magnetic powders or of laminations, with a rectangular prolonged hole (Figure 4.3b), “glued together” with a thermally and mechanically resilient resin.

4.2.1 The Stator Windings

The stator slots are provided with coils connected to form a three-phase winding. The winding of each phase produces an airgap fixed magnetic field with $2p_1$ half-periods per revolution. With D_{is} as the internal stator diameter, the pole pitch τ , that is the half-period of winding magnetomotive force (mmf), is as follows:

$$\tau = \pi D_{is} / 2 p_1 \quad (4.4)$$

The phase windings are phase shifted by $(2/3)\tau$ along the stator periphery and are symmetric. The average number of slots per pole per phase q is

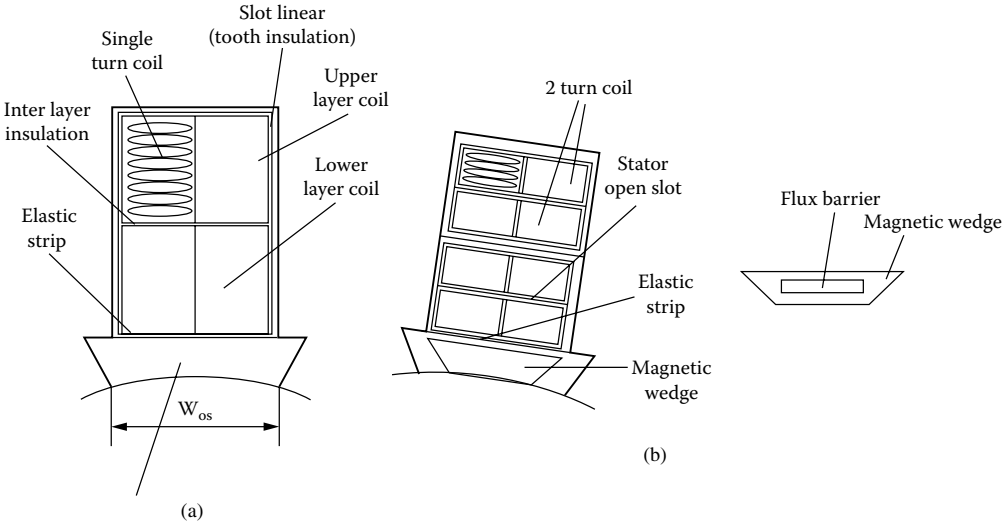


FIGURE 4.3 (a) Stator slotting and (b) magnetic wedge.

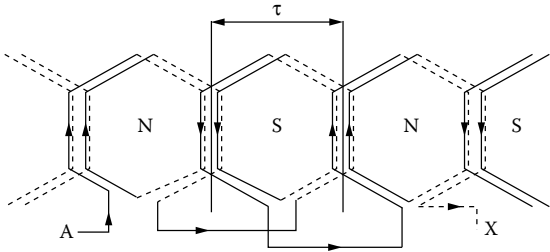


FIGURE 4.4 Lap winding (four poles) with $q = 2$, phase A only.

$$q = \frac{Ns}{2p_1 \cdot 3} \quad (4.5)$$

The number q may be an integer, with a low number of poles ($2p_1 < 8-10$), or it may be a fractionary number:

$$q = a + b/c \quad (4.6)$$

Fractionary q windings are used mainly in SGs with a large number of poles, where a necessarily low integer q ($q \leq 3$) would produce too high a harmonics content in the generator electromagnetic field (emf).

Large and medium power SGs make use of typical lap (multiturn coil) windings (Figure 4.4) or of bar-wave (single-turn coil) windings (Figure 4.5).

The coils of phase A in Figure 4.4 and Figure 4.5 are all in series. A single current path is thus available ($a = 1$). It is feasible to have a current paths in parallel, especially in large power machines (line voltage is generally below 24 kV). With W_{ph} turns in series (per current path), we have the following relationship:

$$N_s = 3 \frac{W_{ph} \cdot a}{n_c} \quad (4.7)$$

with n_c equal to the turns per coil.

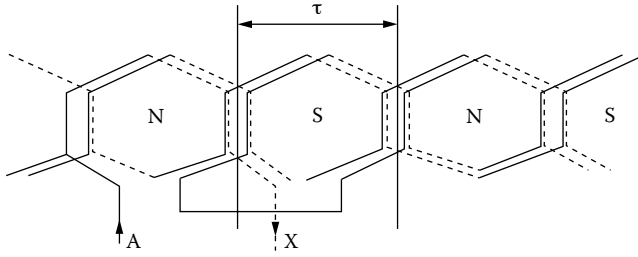


FIGURE 4.5 Basic wave-bar winding with $q = 2$, phase A only.

The coils may be multiturn lap coils or uniturn (bar) type, in wave coils.

A general comparison between the two types of windings (both with integer or fractionary q) reveals the following:

- The multiturn coils ($n_c > 1$) allow for greater flexibility when choosing the number of slots N_s for a given number of current paths a .
- Multiturn coils are, however, manufacturing-wise, limited to 0.3 m long lamination stacks and pole pitches $\tau < 0.8\text{--}1$ m.
- Multiturn coils need bending flexibility, as they are placed with one side in the bottom layer and with the other one in the top layer; bending needs to be done without damaging the electric insulation, which, in turn, has to be flexible enough for the purpose.
- Bar coils are used for heavy currents (above 1500 A). Wave-bar coils imply a smaller number of connectors (Figure 4.5) and, thus, are less costly. The lap-bar coils allow for short pitching to reduce emf harmonics, while wave-bar coils imply 100% average pitch coils.
- To avoid excessive eddy current (skin) effects in deep coil sides, transposition of individual strands is required. In multiturn coils ($n_c \geq 2$), one semi-Roebel transposition is enough, while in single-bar coils, full Roebel transposition is required.
- Switching or lightning strokes along the transmission lines to the SG produce steep-fronted voltage impulses between neighboring turns in the multiturn coil; thus, additional insulation is required. This is not so for the bar (single-turn) coils, for which only interlayer and slot insulation are provided.
- Accidental short-circuit in multiturn coil windings with $a \geq 2$ current path in parallel produce a circulating current between current paths. This unbalance in path currents may be sufficient to trip the pertinent circuit balance relay. This is not so for the bar coils, where the unbalance is less pronounced.
- Though slightly more expensive, the technical advantages of bar (single-turn) coils should make them the favorite solution in most cases.

Alternating current (AC) windings for SGs may be built not only in two layers, but also in one layer. In this latter case, it will be necessary to use 100% pitch coils that have longer end connections, unless bar coils are used.

Stator end windings have to be mechanically supported so as to avoid mechanical deformation during severe transients, due to electrodynamic large forces between them, and between them as a whole and the rotor excitation end windings. As such forces are generally radial, the support for end windings typically looks as shown in Figure 4.6. Note that more on AC winding specifics are included in Chapter 7, which is dedicated to SG design. Here, we derive only the fundamental mmf wave of three-phase stator windings.

The mmf of a single-phase four-pole winding with 100% pitch coils may be approximated with a step-like periodic function if the slot openings are neglected (Figure 4.7). For the case in Figure 4.7 with $q = 2$ and 100% pitch coils, the mmf distribution is rectangular with only one step per half-period. With chorded coils or $q > 2$, more steps would be visible in the mmf. That is, the distribution then better

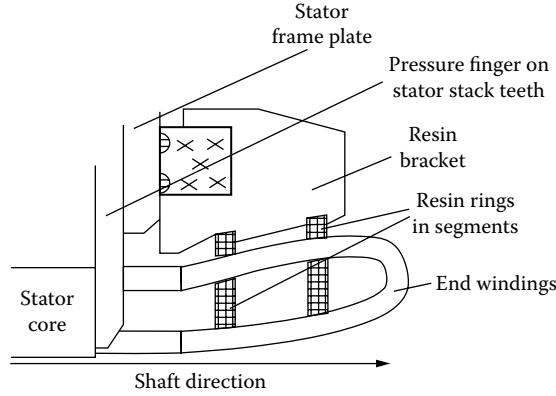


FIGURE 4.6 Typical support system for stator end windings.

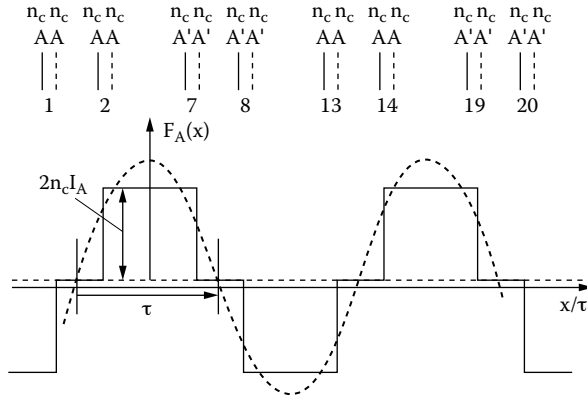


FIGURE 4.7 Stator phase mmf distribution ($2p = 4, q = 2$).

approximates a sinusoid waveform. In general, the phase mmf fundamental distribution for steady state may be written as follows:

$$F_{1A}(x, t) = F_{1m} \cdot \cos \frac{\pi}{\tau} x \cdot \cos \omega_1 t \quad (4.8)$$

$$F_{1m} = 2\sqrt{2} \frac{W_1 K_{w1} I}{\pi p_1} \quad (4.9)$$

where

W_1 = the number of turns per phase in series

I = the phase current (RMS)

p_1 = the number of pole pairs

K_{w1} = the winding factor:

$$K_{w1} = \frac{\sin \pi / 6}{q \cdot \sin \pi / 6q} \cdot \sin \left(\frac{y}{\tau} \frac{\pi}{2} \right) \quad (4.10)$$

with y/τ = coil pitch/pole pitch ($y/\tau > 2/3$).

Equation 4.8 is strictly valid for integer q .

An equation similar to Equation 4.8 may be written for the v^{th} space harmonic:

$$F_{vA}(x, t) = F_{vm} \cos v \frac{\pi}{\tau} x \cos(\omega_1 t)$$

$$F_{vm} = \frac{2\sqrt{2}W_l K_{wv} I}{\pi p_1 v} \quad (4.11)$$

$$K_{wv} = \frac{\sin v\pi/6}{q \cdot \sin(v\pi/6q)} \cdot \sin \frac{y}{\tau} \frac{v\pi}{2} \quad (4.12)$$

Phase B and phase C mmf expressions are similar to Equation 4.8 but with $2\pi/3$ space and time lags. Finally, the total mmf (with space harmonics) produced by a three-phase winding is as follows [2]:

$$F_v(x, t) = \frac{3W_l I \sqrt{2} K_{wv}}{\pi p_1 v} \left[K_{Bl} \cos\left(\frac{v\pi}{\tau} - \omega_1 t - (v-1)\frac{2\pi}{3}\right) - K_{Bll} \cos\left(\frac{v\pi}{\tau} + \omega_1 t - (v+1)\frac{2\pi}{3}\right) \right] \quad (4.13)$$

with

$$K_{Bl} = \frac{\sin(v-1)\pi}{3 \cdot \sin(v-1)\pi/3} \quad (4.14)$$

$$K_{Bll} = \frac{\sin(v+1)\pi}{3 \cdot \sin(v+1)\pi/3}$$

Equation 4.13 is valid for integer q .

For $v = 1$, the fundamental is obtained.

Due to full symmetry, with q integer, only odd harmonics exist. For $v = 1$, $K_{Bl} = 1$, $K_{Bll} = 0$, so the mmf fundamental represents a forward-traveling wave with the following peripheral speed:

$$\frac{dx}{dt} = \frac{\tau \omega_1}{\pi} = 2\tau f_1 \quad (4.15)$$

The harmonic orders are $v = 3K \pm 1$. For $v = 7, 13, 19, \dots$, $dx/dt = 2\tau f_1/v$ and for $v = 5, 11, 17, \dots$, $dx/dt = -2\tau f_1/v$. That is, the first ones are direct-traveling waves, while the second ones are backward-traveling waves. Coil chording ($y/\tau < 1$) and increased q may reduce harmonics amplitude (reduced K_{wv}), but the price is a reduction in the mmf fundamental (K_{w1} decreases).

The rotors of large SGs may be built with salient poles (for $2p_1 > 4$) or with nonsalient poles ($2p_1 = 2, 4$). The solid iron core of the nonsalient pole rotor (Figure 4.8a) is made of 12 to 20 cm thick (axially) rolled steel discs spigoted to each other to form a solid ring by using axial through-bolts. Shaft ends are added (Figure 4.9). Salient poles (Figure 4.8b) may be made of lamination packs tightened axially by through-bolts and end plates and fixed to the rotor pole wheel by hammer-tail key bars.

In general, peripheral speeds around 110 m/sec are feasible only with solid rotors made by forged steel. The field coils in slots (Figure 4.8a) are protected from centrifugal forces by slot wedges that are made either of strong resins or of conducting material (copper), and the end-windings need bandages.

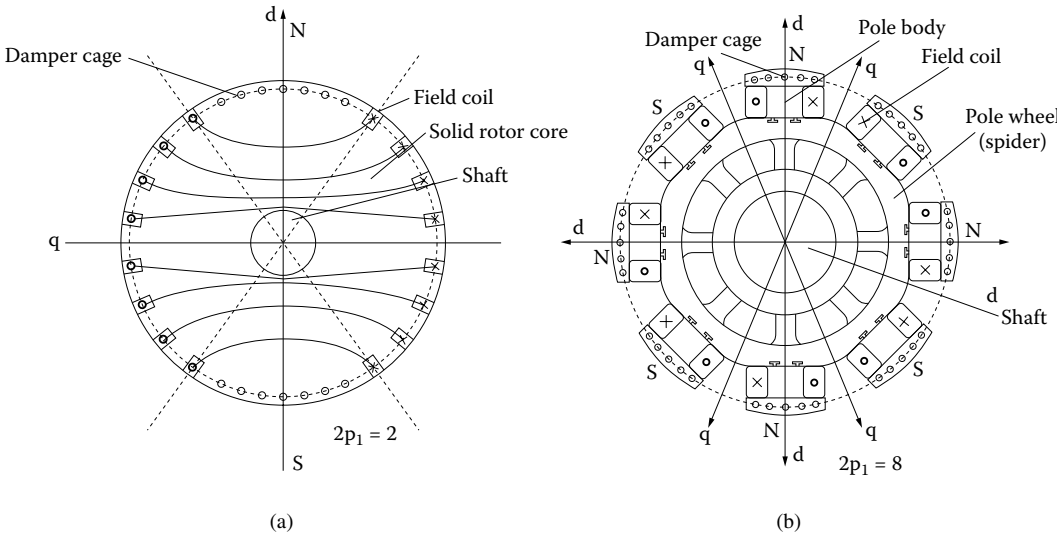


FIGURE 4.8 Rotor configurations: (a) with nonsalient poles $2p_1 = 2$ and (b) with salient poles $2p_1 = 8$.

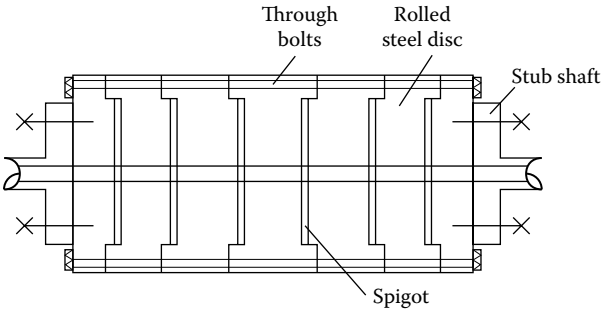


FIGURE 4.9 Solid rotor.

The interpole area in salient pole rotors (Figure 4.8b) is used to mechanically fix the field coil sides so that they do not move or vibrate while the rotor rotates at its maximum allowable speed.

Nonsalient pole (high-speed) rotors show small magnetic anisotropy. That is, the magnetic reluctance of airgap along pole (longitudinal) axis d , and along interpole (transverse) axis q , is about the same, except for the case of severe magnetic saturation conditions.

In contrast, salient pole rotors experience a rather large (1.5 to 1 and more) magnetic saliency ratio between axis d and axis q . The damper cage bars placed in special rotor pole slots may be connected together through end rings (Figure 4.10). Such a complete damper cage may be decomposed in two fictitious cages, one with the magnetic axis along the d axis and the other along the q axis (Figure 4.10), both with partial end rings (Figure 4.10).

4.3 Excitation Magnetic Field

The airgap magnetic field produced by the direct current (DC) field (excitation) coils has a circumferential distribution that depends on the type of the rotor, with salient or nonsalient poles, and on the airgap variation along the rotor pole span. For the time being, let us consider that the airgap is constant under the rotor pole and the presence of stator slot openings is considered through the Carter coefficient K_{C1} , which increases the airgap [2]:

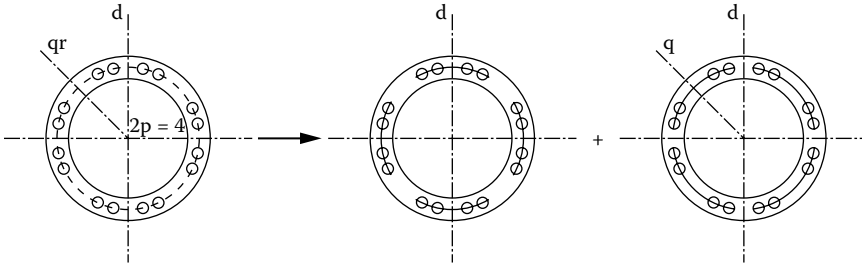


FIGURE 4.10 The damper cage and its d axis and q axis fictitious components.

$$K_{C1} \approx \frac{\tau_s}{\tau_s - \gamma_1 g} > 1, \tau_s - \text{stator_slot_pitch} \quad (4.16)$$

$$\gamma_1 = \frac{4}{\pi} \left[\frac{\left(\frac{W_{os}}{g} \right)}{\tan \left(\frac{W_{os}}{g} \right)} - \ln \sqrt{1 + \left(\frac{W_{os}}{g} \right)^2} \right] \quad (4.17)$$

with W_{os} equal to the stator slot opening and g equal to the airgap.

The flux lines produced by the field coils (Figure 4.11) resemble the field coil mmfs $F_f(x)$, as the airgap under the pole is considered constant (Figure 4.12). The approximate distribution of no-load or field-winding-produced airgap flux density in Figure 4.12 was obtained through Ampere's law.

For salient poles:

$$B_{gFm} = \frac{\mu_0 W_f I_f}{K_c g (1 + K_{s0})}, \text{ for } |x| < \frac{\tau_p}{2} \quad (4.18)$$

and $B_{gFm} = 0$ otherwise (Figure 4.12a).

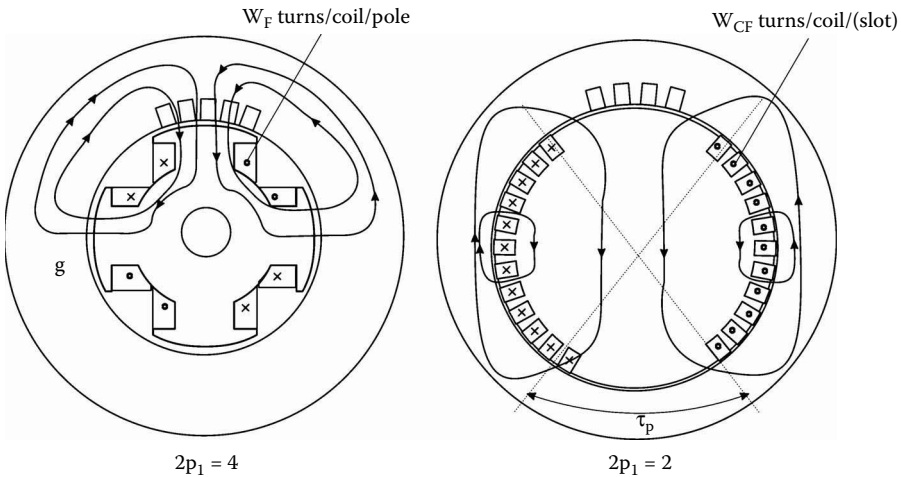


FIGURE 4.11 Basic field-winding flux lines through airgap and stator.

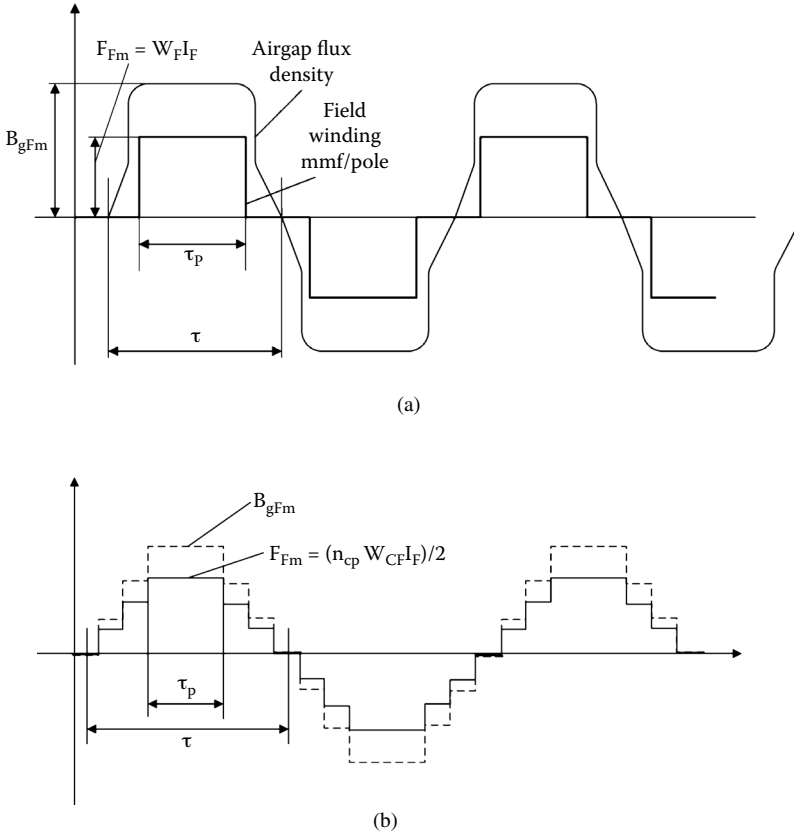


FIGURE 4.12 Field-winding mmf and airgap flux density: (a) salient pole rotor and (b) nonsalient pole rotor.

In practice, $B_{gFm} = 0.6 - 0.8$ T. Fourier decomposition of this rectangular distribution yields the following:

$$B_{gFv}(x) = K_{Fv} \cdot B_{gFm} \cos v \frac{\pi}{\tau} x; \quad v = 1, 3, 5, \dots \quad (4.19)$$

$$K_{Fv} = \frac{4}{\pi} \sin v \frac{\tau_p}{\tau} \frac{\pi}{2} \quad (4.20)$$

Only the fundamental is useful. Both the fundamental distribution ($v = 1$) and the space harmonics depend on the ratio τ_p/τ (pole span/pole pitch). In general, $\tau_p/\tau \approx 0.6 - 0.72$. Also, to reduce the harmonics content, the airgap may be modified (increased), from the pole middle toward the pole ends, as an inverse function of $\cos \pi x/\tau$:

$$g(x) = \frac{g}{\cos \frac{\pi}{\tau} x}, \text{ for } : \frac{-\tau_p}{2} < x < \frac{\tau_p}{2} \quad (4.21)$$

In practice, Equation 4.21 is not easy to generate, but approximations of it, easy to manufacture, are adopted.

Reducing the no-load airgap flux-density harmonics causes a reduction of time harmonics in the stator emf (or no-load stator phase voltage).

For the nonsalient pole rotor (Figure 4.12b):

$$B_{gFm} = \frac{\mu_0 \frac{n_p}{2} W_{CF} I_f}{K_C g (1 + K_{S0})}, \text{ for } |x| < \frac{\tau_p}{2} \quad (4.22)$$

and stepwise varying otherwise (Figure 4.12b). K_{S0} is the magnetic saturation factor that accounts for stator and rotor iron magnetic reluctance of the field paths; n_p – slots per rotor pole.

$$B_{gF0}(x) = K_{F0} \cdot B_{gFM} \cdot \cos \frac{v\pi}{\tau} x \quad (4.23)$$

$$K_{F0} \approx \frac{8}{v^2 \pi^2} \frac{\cos v \frac{\tau_p}{2} \frac{\pi}{2}}{\left(1 - v \frac{\tau_p}{\tau}\right)} \quad (4.24)$$

It is obvious that, in this case, the flux density harmonics are lower; thus, constant airgap (cylindrical rotor) is feasible in all practical cases.

Let us consider only the fundamentals of the no-load flux density in the airgap:

$$B_{gF1}(x_r) = B_{gFm1} \cos \frac{\pi}{\tau} x_r \quad (4.25)$$

For constant rotor speed, the rotor coordinate x_r is related to the stator coordinate x_s as follows:

$$\frac{\pi}{\tau} x_r = \frac{\pi}{\tau} x_s - \omega_r t - \theta_0 \quad (4.26)$$

The rotor rotates at angular speed ω_r (in electrical terms: $\omega_r = p_1 \Omega_r - \Omega_r$, mechanical angular velocity). θ_0 is an arbitrary initial angle; let $\theta_0 = 0$.

With Equation 4.26, Equation 4.25 becomes

$$B_{gF1}(x_s, t) = B_{gFm1} \cos \left(\frac{\pi}{\tau} x_s - \omega_r t \right) \quad (4.27)$$

So, the excitation airgap flux density represents a forward-traveling wave at rotor speed. This traveling wave moves in front of the stator coils at the tangential velocity u_s :

$$u_s = \frac{dx_s}{dt} = \frac{\tau \omega_r}{\pi} \quad (4.28)$$

It is now evident that, with the rotor driven by a prime mover at speed ω_r , and the stator phases open, the excitation airgap magnetic field induces an emf in the stator windings:

$$E_{A1}(t) = -\frac{d}{dt} W_1 K_{W1} \int_{-\frac{\tau}{2}}^{+\frac{\tau}{2}} l_{stack} B_{gF1}(x_s, t) dx_s \quad (4.29)$$

and, finally,

$$E_{A1}(t) = E_1 \sqrt{2} \cos \omega_r t \quad (4.30)$$

$$E_{1m} = \pi \sqrt{2} \left(\frac{\omega_r}{2\pi} \right) B_{gFm1} l_{stack} W_1 K_{W1} \frac{\tau^2}{\pi} \quad (4.31)$$

with l_{stack} equal to the stator stack length.

As the three phases are fully symmetric, the emfs in them are as follows:

$$E_{A,B,C,1}(t) = E_{1m} \sqrt{2} \cos \left[\omega_r t - (i-1) \frac{2\pi}{3} \right] \quad (4.32)$$

$i = 1, 2, 3$

So, we notice that the excitation coil currents in the rotor are producing at no load (open stator phases) three symmetric emfs with frequency ω_r that is given by the rotor speed $\Omega_r = \omega_r/p_1$.

4.4 The Two-Reaction Principle of Synchronous Generators

Let us now suppose that an excited SG is driven on no load at speed ω_r . When a balanced three-phase load is connected to the stator (Figure 4.13a), the presence of emfs at frequency ω_r will naturally produce currents of the same frequency. The phase shift between the emfs and the phase current ψ is dependent on load nature (power factor) and on machine parameters, not yet mentioned (Figure 4.13b). The sinusoidal emfs and currents are represented as simple phasors in Figure 4.13b. Because of the magnetic anisotropy of the rotor along axes d and q , it helps to decompose each phase current into two components: one in phase with the emf and the other one at 90° with respect to the former: I_{Aq} , I_{Bq} , I_{Cq} , and, respectively, I_{Ad} , I_{Bd} , I_{Cd} .

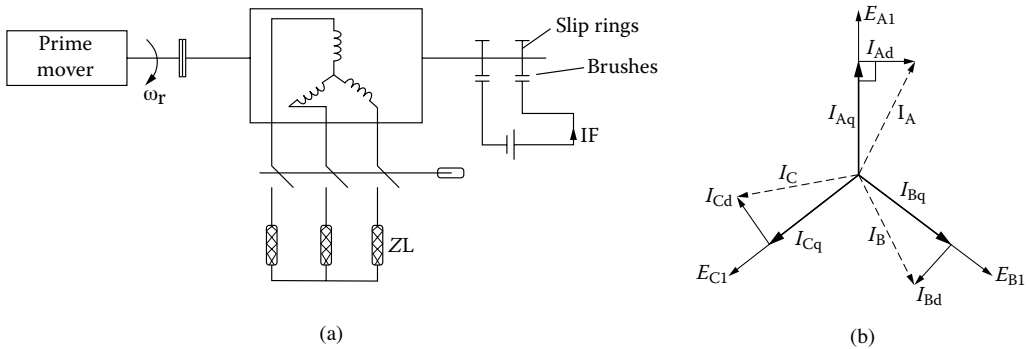


FIGURE 4.13 Illustration of synchronous generator principle: (a) the synchronous generator on load and (b) the emf and current phasors.

As already proven in the paragraph on windings, three-phase symmetric windings flowed by balanced currents of frequency ω_r will produce traveling mmfs (Equation 4.13):

$$F_d(x, t) = -F_{dm} \cos\left(\frac{\pi}{\tau} x_s - \omega_r t\right) \quad (4.33)$$

$$F_{dm} = \frac{3\sqrt{2}I_d W_1 K_{W1}}{\pi p_1}; I_d = |I_{Ad}| = |I_{Bd}| = |I_{Cd}| \quad (4.34)$$

$$F_q(x, t) = F_{qm} \cos\left(\frac{\pi}{\tau} x_s - \omega_r t - \frac{\pi}{2}\right) \quad (4.35)$$

$$F_{qm} = \frac{3\sqrt{2}I_q W_1 K_{W1}}{\pi p_1}; I_q = |I_{Aq}| = |I_{Bq}| = |I_{Cq}| \quad (4.36)$$

In essence, the d -axis stator currents produce an mmf aligned to the excitation airgap flux density wave (Equation 4.26) but opposite in sign (for the situation in [Figure 4.13b](#)). This means that the d -axis mmf component produces a magnetic field “fixed” to the rotor and flowing along axis d as the excitation field does.

In contrast, the q -axis stator current components produce an mmf with a magnetic field that is again “fixed” to the rotor but flowing along axis q .

The emfs produced by motion in the stator windings might be viewed as produced by a fictitious three-phase AC winding flowed by symmetric currents $\underline{I}_{FA}$, $\underline{I}_{FB}$, $\underline{I}_{FC}$ of frequency ω_r :

$$\underline{E}_{A,B,C} = -j\omega_r M_{FA} \underline{I}_{FA,B,C} \quad (4.37)$$

From what we already discussed in this paragraph,

$$E_A(t) = 2\pi \frac{\omega_r}{2\pi} K_{W1} \frac{2}{\pi} l_{stack} \tau \cdot \frac{\mu_0 W_F I_f K_{F1}}{K_c g(1 + K_{S0})} \cos \omega_{rt} \quad (4.38)$$

$$W_F = \frac{n_p}{2} W_{CF}; \text{ for nonsalient pole rotor, see (4.22)}$$

The fictitious currents $\underline{I}_{FA}$, $\underline{I}_{FB}$, $\underline{I}_{FC}$ are considered to have the root mean squared (RMS) value of I_f in the real field winding. From Equation 4.37 and Equation 4.38:

$$M_{FA} = \mu_0 \frac{\sqrt{2}}{\pi} \frac{W_1 W_F K_{W1} \tau \cdot l_{stack}}{K_c g(1 + K_{S0})} K_{F1} \quad (4.39)$$

M_{FA} is called the mutual rotational inductance between the field and armature (stator) phase windings.

The positioning of the fictitious $\underline{I}_f$ (per phase) in the phasor diagram (according to Equation 4.37) and that of the stator phase current phasor $\underline{I}$ (in the first or second quadrant for generator operation and in the third or fourth quadrant for motor operation) are shown in [Figure 4.14](#).

The generator–motor divide is determined solely by the electromagnetic (active) power:

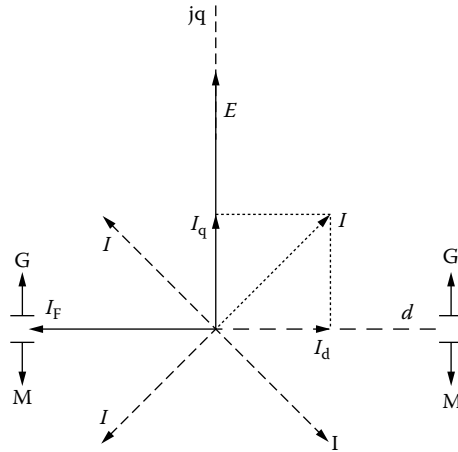


FIGURE 4.14 Generator and motor operation modes.

$$P_{elm} = 3 \operatorname{Re} \left(\underline{E} \cdot \underline{I}^* \right) > 0 \text{ generator, } < 0 \text{ motor} \quad (4.40)$$

The reactive power Q_{elm} is

$$Q_{elm} = 3 \operatorname{Im} \operatorname{ag} \left(\underline{E} \cdot \underline{I}^* \right) < > 0 (\text{generator / motor}) \quad (4.41)$$

The reactive power may be either positive (delivered) or negative (drawn) for both motor and generator operation.

For reactive power “production,” I_d should be opposite from I_F , that is, the longitudinal armature reaction airgap field will oppose the excitation airgap field. It is said that only with demagnetizing longitudinal armature reaction — machine overexcitation — can the generator (motor) “produce” reactive power. So, for constant active power load, the reactive power “produced” by the synchronous machine may be increased by increasing the field current I_F . On the contrary, with underexcitation, the reactive power becomes negative; it is “absorbed.” This extraordinary feature of the synchronous machine makes it suitable for voltage control, in power systems, through reactive power control via I_F control. On the other hand, the frequency ω_r , tied to speed, $\Omega_r = \omega_r / p_1$, is controlled through the prime mover governor, as discussed in Chapter 3. For constant frequency power output, speed has to be constant. This is so because the two traveling fields — that of excitation and, respectively, that of armature windings — interact to produce constant (nonzero-average) electromagnetic torque only at standstill with each other.

This is expressed in Equation 4.40 by the condition that the frequency of $\underline{E}_1 - \omega_r$ — be equal to the frequency of stator current $I_1 - \omega_1 = \omega_r$ — to produce nonzero active power. In fact, Equation 4.40 is valid only when $\omega_r = \omega_1$, but in essence, the average instantaneous electromagnetic power is nonzero only in such conditions.

4.5 The Armature Reaction Field and Synchronous Reactances

As during steady state magnetic field waves in the airgap that are produced by the rotor (excitation) and stator (armature) are relatively at standstill, it follows that the stator currents do not induce voltages (currents) in the field coils on the rotor. The armature reaction (stator) field wave travels at rotor speed; the longitudinal I_{aA} , I_{aB} , I_{aC} and transverse I_{qA} , I_{qB} , I_{qC} armature current (reaction) fields are fixed to the

rotor: one along axis d and the other along axis q . So, for these currents, the machine reacts with the magnetization reluctances of the airgap and of stator and rotor iron with no rotor-induced currents.

The trajectories of armature reaction d and q fields and their distributions are shown in Figure 4.15a, Figure 4.15b, Figure 4.16a, and Figure 4.16b, respectively. The armature reaction mmfs F_{d1} and F_{q1} have a sinusoidal space distribution (only the fundamental reaction is considered), but their airgap flux densities do not have a sinusoidal space distribution. For constant airgap zones, such as it is under the constant airgap salient pole rotors, the airgap flux density is sinusoidal. In the interpole zone of a salient pole machine, the equivalent airgap is large, and the flux density decreases quickly (Figure 4.15 and Figure 4.16).

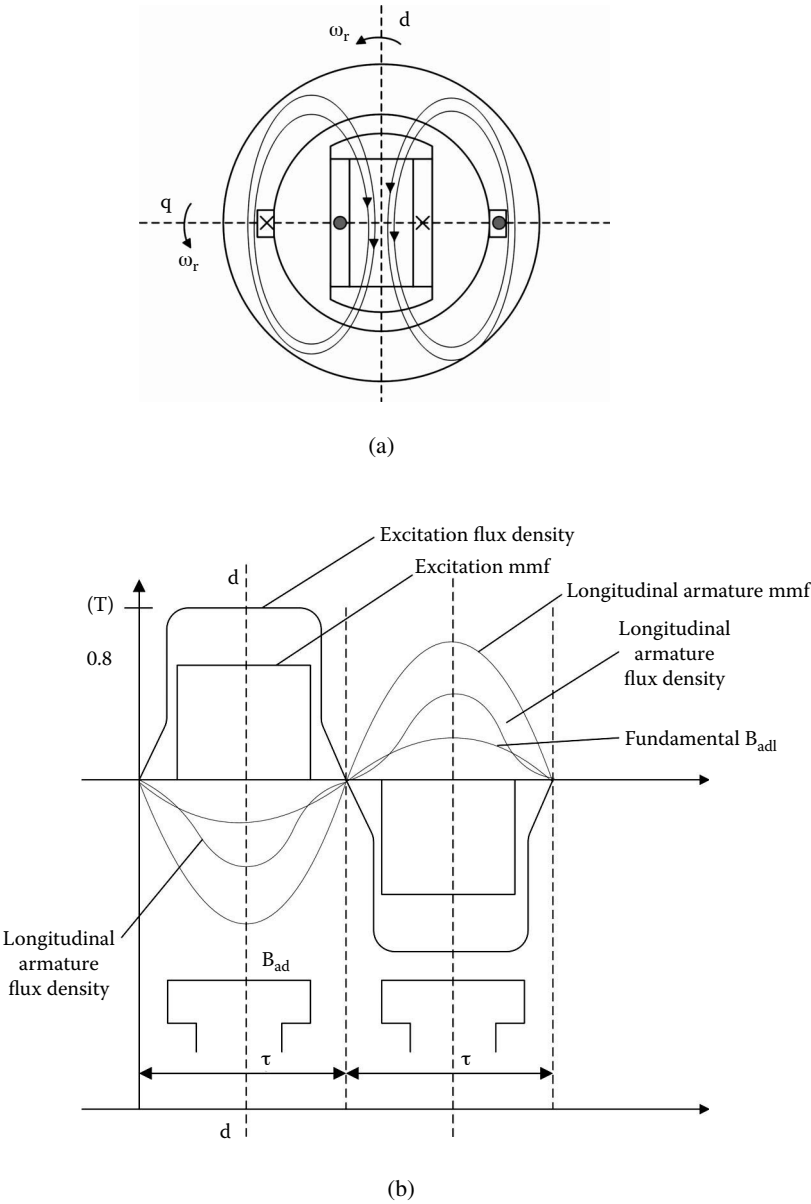


FIGURE 4.15 Longitudinal (d axis) armature reaction: (a) armature reaction flux paths and (b) airgap flux density and mmfs.

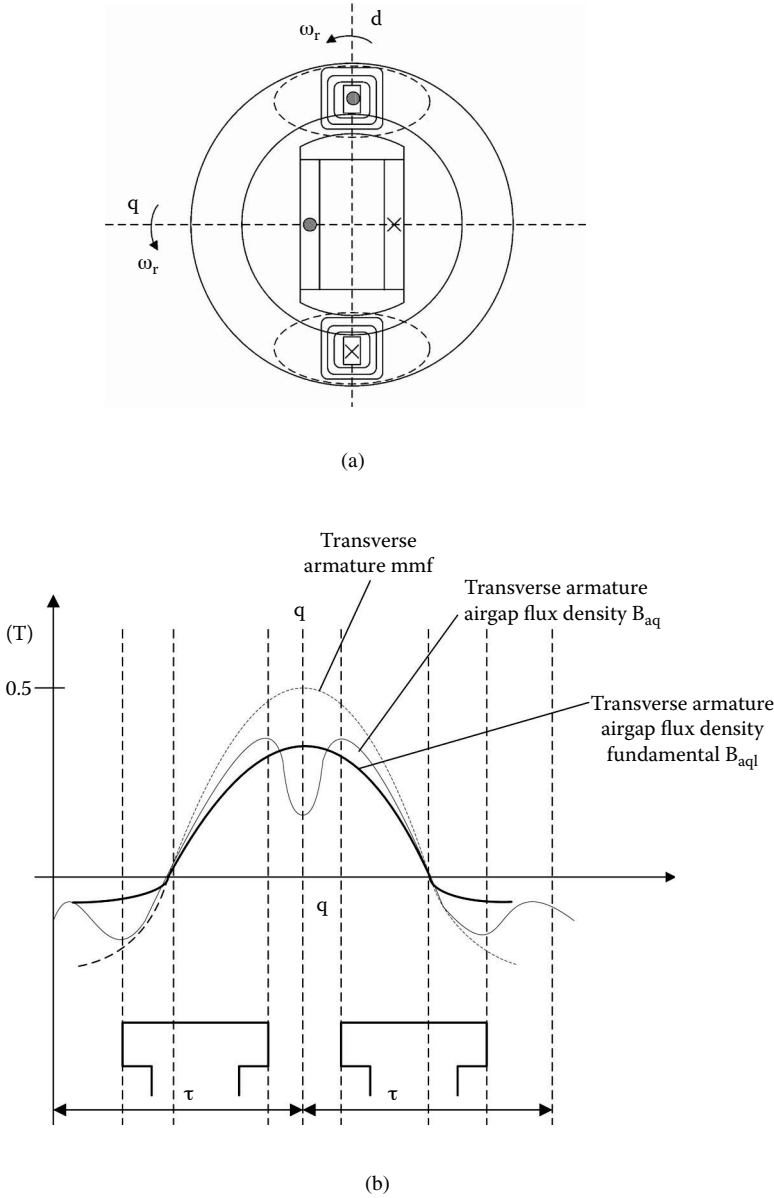


FIGURE 4.16 Transverse (q axis) armature reaction: (a) armature reaction flux paths and (b) airgap flux density and mmf.

Only with the finite element method (FEM) can the correct flux density distribution of armature (or excitation, or combined) mmfs be computed. For the time being, let us consider that for the d axis mmf, the interpolar airgap is infinite, and for the q axis mmf, it is $g_q = 6g$. In axis q , the transverse armature mmf is at maximum, and it is not practical to consider that the airgap in that zone is infinite, as that would lead to large errors. This is not so for d axis mmf, which is small toward axis q , and the infinite airgap approximate is tolerable.

We should notice that the q -axis armature reaction field is far from a sinusoid. This is so only for salient pole rotor SGs. Under steady state, however, we operate only with fundamentals, and with respect to them, we define the reactances and other variables. So, we now extract the fundamentals of B_{ad} and B_{aq} to find the B_{ad1} and B_{aq1} :

$$B_{ad1} = \frac{1}{\tau} \int_0^{\tau} B_{ad}(x_r) \sin\left(\frac{\pi}{\tau} x_r\right) dx_r \quad (4.42)$$

with

$$B_{ad} = 0, \text{ for } : 0 < x_r < \frac{\tau - \tau_p}{2} \text{ and } \frac{\tau + \tau_p}{2} < x < \tau \quad (4.43)$$

$$B_{ad} = \frac{\mu_0 F_{dm} \sin \frac{\pi}{\tau} x_r}{K_c g (1 + K_{sd})} \text{ for } \frac{\tau - \tau_p}{2} < x_r < \frac{\tau + \tau_p}{2}$$

and finally,

$$B_{ad1} = \frac{\mu_0 F_{dm} K_{d1}}{K_c g (1 + K_{sd})}; K_{d1} \approx \frac{\tau_p}{\tau} + \frac{1}{\pi} \sin \frac{\tau_p}{\tau} \pi \quad (4.44)$$

In a similar way,

$$B_{aq} = \frac{\mu_0 F_{qm} \sin \frac{\pi}{\tau} x_r}{K_c g (1 + K_{sq})}, \text{ for } : 0 \leq x_r \leq \frac{\tau_p}{2} \text{ and } \frac{\tau + \tau_p}{2} < x < \tau \quad (4.45)$$

$$B_{aq} = \frac{\mu_0 F_{qm} \sin \frac{\pi}{\tau} x_r}{K_c g_q (1 + K_{sq})} \text{ for } \frac{\tau_p}{2} < x_r < \frac{\tau + \tau_p}{2}$$

$$B_{aq1} = \frac{\mu_0 F_{qm} K_{q1}}{K_c g}; \quad (4.46)$$

$$K_{q1} = \frac{\tau_p}{\tau} - \frac{1}{\pi} \sin \frac{\tau_p}{\tau} \pi + \frac{2}{3\pi} \cos\left(\frac{\tau_p}{\tau} \frac{\pi}{2}\right)$$

Notice that the integration variable was x_p referring to rotor coordinates.

Equation 4.44 and Equation 4.46 warrant the following remarks:

- The fundamental armature reaction flux density in axes d and q are proportional to the respective stator mmfs and inversely proportional to airgap and magnetic saturation equivalent factors K_{sd} and K_{sq} (typically, $K_{sd} \neq K_{sq}$).
- B_{ad1} and B_{aq1} are also proportional to equivalent armature reaction coefficients K_{d1} and K_{q1} . Both smaller than unity ($K_{d1} < 1, K_{q1} < 1$), they account for airgap nonuniformity (slotting is considered only by the Carter coefficient). Other than that, B_{ad1} and B_{aq1} formulae are similar to the airgap flux density fundamental B_{a1} in a uniform airgap machine with same stator, B_{a1} :

$$B_{a1} = \frac{\mu_0 F_1}{K_c g (1 + K_s)}; F_1 = \frac{3\sqrt{2} W_l K_{w1} I_l}{\pi p_1} \quad (4.47)$$

The cyclic magnetization inductance X_m of a uniform airgap machine with a three-phase winding is straightforward, as the self-emf in such a winding, E_{a1} , is as follows:

$$E_{a1} = \omega_r W_1 K_{W1} \Phi_{a1}; \Phi_{a1} = \frac{2}{\pi} B_{a1} \tau \cdot l_{stack} \quad (4.48)$$

From Equation 4.47 and Equation 4.48, X_m is

$$X_m = \frac{E_{a1}}{I_1 \sqrt{2}} = \frac{6\mu_0 \omega_r}{\pi^2} \frac{(W_1 K_{W1})^2 \tau \cdot l_{stack}}{K_C g (1 + K_S)} \quad (4.49)$$

It follows logically that the so-called cyclic magnetization reactances of synchronous machines X_{dm} and X_{qm} are proportional to their flux density fundamentals:

$$X_{dm} = X_m \frac{B_{ad1}}{B_{a1}} = X_m \cdot K_{d1} \quad (4.50)$$

$$X_{qm} = X_m \frac{B_{aq1}}{B_{a1}} = X_m K_{q1} \quad (4.51)$$

and, $K_{sd} = K_{sq} = K_s$ was implied.

The term “cyclic” comes from the fact that these reactances manifest themselves only with balanced stator currents and symmetric windings and only for steady state. During steady state with balanced load, the stator currents manifest themselves by two distinct magnetization reactances, one for axis d and one for axis q , acted upon by the d and q phase current components. We should add to these the leakage reactance typical to any winding, X_{l1} , to compose the so-called synchronous reactances of the synchronous machine (X_d and X_q):

$$X_d = X_{l1} + X_{dm} \quad (4.52)$$

$$X_q = X_{l1} + X_{qm} \quad (4.53)$$

The damper cage currents are zero during steady state with balanced load, as the armature reaction field components are at standstill with the rotor and have constant amplitudes (due to constant stator current amplitude).

We are now ready to proceed with SG equations for steady state under balanced load.

4.6 Equations for Steady State with Balanced Load

We previously introduced stator fictitious AC three-phase field currents $\underline{I}_{F,A,B,C}$ to emulate the field-winding motion-produced emfs in the stator phases $\underline{E}_{A,B,C}$. The decomposition of each stator phase current $\underline{I}_{qA,B,C}$, $\underline{I}_{dA,B,C}$, which then produces the armature reaction field waves at standstill with respect to the excitation field wave, has led to the definition of cyclic synchronous reactances X_d and X_q . Consequently, as our fictitious machine is under steady state with zero rotor currents, the per phase equations in complex (phasors) are simply as follows:

$$\underline{I}_1 R_1 + \underline{V}_1 = \underline{E}_1 - jX_d \underline{I}_d - jX_q \underline{I}_q \quad (4.54)$$

$$\underline{E} = -jX_{Fm} \times \underline{I}_F; X_{Fm} = \omega_r M_{FA} \quad (4.55)$$

$$\underline{I}_1 = \underline{I}_d + \underline{I}_q$$

RMS values all over in Equation 4.54 and Equation 4.55.

To secure the correct phasing of currents, let us consider $\underline{I}_F$ along axis d (real). Then, according to Figure 4.13,

$$\underline{I}_q = I_q \times \left(-j \frac{\underline{I}_F}{I_F} \right); \underline{I}_d = I_d \frac{\underline{I}_F}{I_F}; I_1 = \sqrt{I_d^2 + I_q^2} \quad (4.56)$$

With $I_F > 0$, I_d is positive for underexcitation ($E_1 < V_1$) and negative for overexcitation ($E_1 > V_1$). Also, I_q in Equation 4.56 is positive for generating and negative for motoring.

The terminal phase voltage $\underline{V}_1$ may represent the power system voltage or an independent load $\underline{Z}_L$:

$$\underline{Z}_L = \frac{\underline{V}_1}{\underline{I}_1} \quad (4.57)$$

A power system may be defined by an equivalent internal emf $\underline{E}_{ps}$ and an internal impedance $\underline{Z}_0$:

$$\underline{V}_1 = \underline{E}_{ps} + \underline{Z}_{ps} \underline{I}_1 \quad (4.58)$$

For an infinite power system, $|\underline{Z}_{ps}| = 0$ and $\underline{E}_{ps}$ is constant. For a limited power system, either only $|\underline{Z}_{ps}| \neq 0$, or also E_{ps} varies in amplitude, phase, or frequency. The power system impedance $\underline{Z}_{ps}$ includes the impedance of multiple generators in parallel, of transformers, and of power transmission lines.

The power balance applied to Equation 4.54, after multiplication by $3I_1^*$, yields the following:

$$P_1 + jQ_1 = 3\underline{V}_1 \underline{I}_1^* = 3\underline{E}_1 \underline{I}_1^* - 3(\underline{I}_1)^2 R_1 - 3jX_{1l}(\underline{I}_1)^2 - 3j(X_{dm} \underline{I}_d + X_{qm} \underline{I}_q) \underline{I}_1^* \quad (4.59)$$

The real part represents the active output power P_1 , and the imaginary part is the reactive power, both positive if delivered by the SG:

$$P_1 = 3E_1 I_q - 3I_1^2 R_1 + 3(X_{dm} - X_{qm}) I_d I_q = 3V_1 I_1 \cos \phi_1 \quad (4.60)$$

$$Q_1 = -3E_1 I_d - 3I_1^2 X_{sl} - 3(X_{dm} I_d^2 + X_{qm} I_q^2) = 3V_1 I_1 \sin \phi_1 \quad (4.61)$$

As seen from Equation 4.60 and Equation 4.61, the active power is positive (generating) only with $I_q > 0$. Also, with $X_{dm} \geq X_{qm}$, the anisotropy active power is positive (generating) only with positive I_d (magnetization armature reaction along axis d). But, positive I_d in Equation 4.61 means definitely negative (absorbed) reactive power, and the SG is underexcited.

In general, $X_{dm}/X_{qm} = 1.0\text{--}1.7$ for most SGs with electromagnetic excitation. Consequently, the anisotropy electromagnetic power is notably smaller than the interaction electromagnetic power. In nonsalient pole machines, $X_{dm} \approx (1.01\text{--}1.05)X_{qm}$ due to the presence of rotor slots in axis q that increase the equivalent airgap (K_C increases due to double slotting). Also, when the SG saturates (magnetically), the level of saturation under load may be, in some regimes, larger than in axis d . In other regimes, when magnetic saturation is larger in axis d , a nonsalient pole rotor may have a slight inverse magnetic saliency ($X_{dm} <$

X_{qm}). As only the stator winding losses have been considered ($3R_1 I_1^2$), the total electromagnetic power P_{elm} is as follows:

$$P_{elm} = 3E_1 I_q + 3(X_{dm} - X_{qm}) I_d I_q \quad (4.62)$$

Now, the electromagnetic torque T_e is

$$T_e = \frac{P_{elm}}{\frac{\omega_r}{p_1}} = 3p_1 \left[M_{fA} I_F I_q + (L_{dm} - L_{qm}) I_d I_q \right] \quad (4.63)$$

with

$$L_{dm} = X_{dm} / \omega_r; L_{qm} = X_{qm} / \omega_r \quad (4.64)$$

And, from Equation 4.37,

$$E_1 = \omega_r M_{FA} I_F \quad (4.65)$$

We may also separate in the stator phase flux linkage Ψ_1 , the two components Ψ_d and Ψ_q :

$$\begin{aligned} \Psi_d &= M_{FA} + L_d I_d \\ \Psi_q &= L_q I_q \end{aligned} \quad (4.66)$$

$$L_d = \frac{X_d}{\omega_r}; L_q = \frac{X_q}{\omega_r} \quad (4.67)$$

The total stator phase flux linkage Ψ_1 is

$$\Psi_1 = \sqrt{\Psi_d^2 + \Psi_q^2}; I_1 = \sqrt{I_d^2 + I_q^2} \quad (4.68)$$

As expected, from Equation 4.63, the electromagnetic torque does not depend on frequency (speed) ω_p , but only on field current and stator current components, besides the machine inductances: the mutual one, M_{FA} , and the magnetization ones L_{dm} and L_{qm} . The currents I_F , I_d , I_q influence the level of magnetic saturation in stator and rotor cores, and thus M_{FA} , L_{dm} , and L_{qm} are functions of all of them.

Magnetic saturation is an involved phenomenon that will be treated in [Chapter 5](#).

The shaft torque T_a differs from electromagnetic torque T_e by the mechanical power loss (p_{mec}) braking torque:

$$T_a = T_e + \frac{p_{mec}}{(\omega_r / p_1)} \quad (4.69)$$

For generator operation mode, T_e is positive, and thus, $T_a > T_e$. Still missing are the core losses located mainly in the stator.

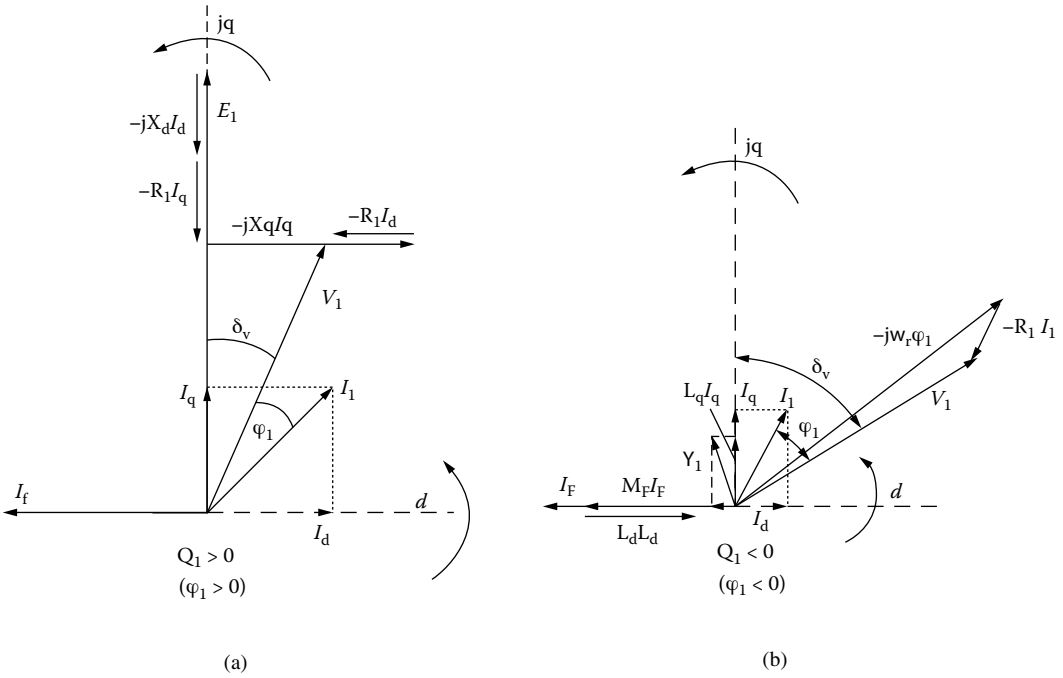


FIGURE 4.17 Phasor diagrams: (a) standard and (b) modified but equivalent.

4.7 The Phasor Diagram

Equation 4.54, Equation 4.55, and Equation 4.66 through Equation 4.68 lead to a new voltage equation:

$$\underline{I}_1 R_1 + \underline{V}_1 = -j\omega_r \underline{\Psi}_1 = \underline{E}_t; \underline{\Psi}_1 = \underline{\Psi}_d + j\underline{\Psi}_q \quad (4.70)$$

where $\underline{E}_t$ is total flux phase emf in the SG. Now, two phasor diagrams, one suggested by Equation 4.54 and one by Equation 4.70 are presented in Figure 4.17a and Figure 4.17b, respectively.

The time phase angle δ_v between the emf $\underline{E}_t$ and the phase voltage $\underline{V}_1$ is traditionally called the internal (power) angle of the SG. As we wrote Equation 4.54 and Equation 4.70 for the generator association of signs, $\delta_v > 0$ for generating ($I_q > 0$) and $\delta_v < 0$ for motoring ($I_q < 0$).

For large SGs, even the stator resistance may be neglected for more clarity in the phasor diagrams, but this is done at the price of “losing” the copper loss consideration.

4.8 Inclusion of Core Losses in the Steady-State Model

The core loss due to the fundamental component of the magnetic field wave produced by both excitation and armature mmf occurs only in the stator. This is so because the two field waves travel at rotor speed. We may consider, to a first approximation, that the core losses are related directly to the main (airgap) magnetic flux linkage $\underline{\Psi}_{1m}$:

$$\underline{\Psi}_{1m} = M_{FA} \underline{I}_F + L_{dm} \underline{I}_d + L_{qm} \underline{I}_q = \underline{\Psi}_{dm} + j\underline{\Psi}_{qm} \quad (4.71)$$

$$\underline{\Psi}_{dm} = M_{FA} I_F + L_{dm} I_d; \underline{\Psi}_{qm} = L_{qm} I_q \quad (4.72)$$

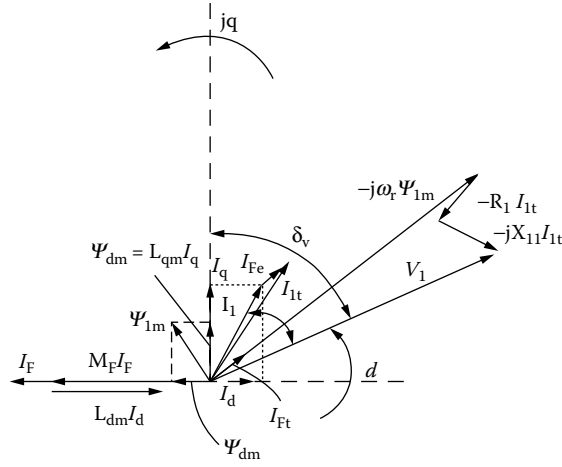


FIGURE 4.18 Phasor diagram with core loss included.

The leakage flux linkage components $L_{sl}I_d$ and $L_{sl}I_q$ do not produce significant core losses, as $L_{sl}/L_{dm} < 0.15$ in general, and most of the leakage flux lines flow within air zones (slot, end windings, airgap).

Now, we will consider a fictitious three-phase stator short-circuited resistive-only winding, R_{Fe} which accounts for the core loss. Neglecting the reaction field of core loss currents I_{Fe} , we have the following:

$$-\frac{d\Psi_{1m0}}{dt} = R_{Fe} I_{Fe} = -j\omega_r \Psi_{1m0} \quad (4.73)$$

R_{Fe} is thus “connected” in parallel to the main flux emf ($-j\omega_r \Psi_{1m}$). The voltage equation then becomes

$$I_{1t} (R_l + jX_d) + V_1 = -j\omega_l \Psi_{1m} \quad (4.74)$$

with

$$I_{1t} = I_d + I_q + I_{Fe} = I_1 + I_{Fe} \quad (4.75)$$

The new phasor diagram of Equation 4.74 is shown in Figure 4.18.

Though core losses are small in large SGs and do not change the phasor diagram notably, their inclusion allows for a correct calculation of efficiency (at least at low loads) and of stator currents as the power balance yields the following:

$$P_1 = 3V_1 I_{1t} \cos \phi_1 = 3\omega_r M_{FA} I_F I_q + 3\omega_r (L_{dm} - L_{qm}) I_d I_q - 3R_l I_{1t}^2 - 3 \frac{\omega_r^2 \Psi_{1m}^2}{R_{Fe}} \quad (4.76)$$

$$\Psi_{1m} = M_{FA} I_f + L_{dm} I_d - jL_{qm} I_q \quad (4.77)$$

$$I_{Fe} = \frac{-j\omega_r \Psi_{1m}}{R_{Fe}}; I_{1t} = I_d + I_q + I_{Fe} \quad (4.78)$$

Once the SG parameters R_1 , R_{Fe} , L_{dm} , L_{qm} , M_{FA} , excitation current I_F , speed (frequency) — $\omega_r/p_1 = 2\pi n$ (rps) — are known, the phasor diagram in Figure 4.17 allows for the computation of I_d , I_q , provided the power angle δ_v and the phase voltage V_1 are also given. After that, the active and reactive power delivered by the SG may be computed. Finally, the efficiency η_{SG} is as follows:

$$\eta_{SG} = \frac{P_1}{P_1 + p_{Fe} + p_{copper} + p_{mec} + p_{add}} = \frac{P_{elm} - p_{copper} - p_{Fe} - p_{add}}{P_{elm} + p_{mec}} \quad (4.79)$$

with p_{add} equal to additional losses on load.

Alternatively, with I_F as a parameter, I_d and I_q can be modified (given) such that $\sqrt{I_d^2 + I_q^2} = I_1$ can be given as a fraction of full load current. Note that while decades ago, the phasor diagrams were used for graphical computation of performance, nowadays they are used only to illustrate performance and derive equations for a pertinent computer program to calculate the same performance faster and with increased precision.

Example 4.1

The following data are obtained from a salient pole rotor synchronous hydrogenerator: $S_N = 72$ MVA, $V_{line} = 13$ kV/star connection, $2p_1 = 90$, $f_1 = 50$ Hz, $q_1 =$ three slots/pole/phase, $I_{lr} = 3000$ A, $R_1 = 0.0125 \Omega$, $(\eta_r)_{\cos 1=1} = 0.9926$, and $p_{Fen} = p_{mec}$. Additional data are as follows: stator interior diameter $D_{is} = 13$ m, stator active stack length $l_{stack} = 1.4$ m, constant airgap under the poles $g = 0.020$ m, Carter coefficient $K_C = 1.15$, and $\tau_p/\tau = 0.72$. The equivalent unique saturation factor $K_s = 0.2$.

The number of turns in series per phase is $W_1 = p_1 q_1 \times \text{one turn/coil} = 45 \times 3 \times 1 = 115$ turns/phase.

Let us calculate the following:

1. The stator winding factor K_{W1}
2. The d and q magnetization reactances X_{dm} , X_{qm}
3. X_d , X_q , with $X_{ll} = 0.2X_{dm}$
4. Rated core and mechanical losses P_{Fen} , p_{mec}
5. x_d , x_q , r_1 in P.U. with $Z_n = V_{lph}/I_{lr}$
6. E_1 , I_d , I_q , I_1 , E_1 , P_1 , Q_1 , by neglecting all losses at $\cos \psi_1 = 1$ and $\delta_v = 30^\circ$
7. The no-load airgap flux density ($K_s = 0.2$) and the corresponding rotor-pole mmf $W_F I_F$

Solution:

1. The winding factor K_{W1} (Equation 4.10) is as follows:

$$K_{W1} = \frac{\sin \pi / 6}{3 \sin(\pi / 6 \cdot 3)} \sin\left(\frac{1}{1} \cdot \frac{\pi}{2}\right) = 0.9598$$

Full pitch coils are required ($y/\tau = 1$), as the single-layer case is considered.

2. The expressions of X_{dm} and X_{qm} are shown in Equation 4.49 through Equation 4.51:

$$X_{dm} = X_m \cdot K_{d1}$$

$$X_{qm} = X_m \cdot K_{q1}$$

From Equation 4.44,

$$K_{d1} = \frac{\tau_p}{\tau} + \frac{1}{\pi} \sin \frac{\tau_p}{\tau} \pi = 0.72 + \frac{1}{\pi} \sin 0.72 \cdot \pi = 0.96538$$

$$K_{q1} = \frac{\tau_p}{\tau} - \frac{1}{\pi} \sin \frac{\tau_p}{\tau} \pi + \frac{2}{3\pi} \cos \frac{\tau_p}{\tau} \frac{\pi}{2} = 0.4776 + 0.0904 = 0.565$$

$$X_m = \frac{6\mu_0}{\pi^2} \omega_r \frac{(W_1 K_{W1})^2 \cdot \tau \cdot l_{stack}}{K_C g (1 + K_s) p_1}$$

$$\text{with: } \tau = \pi D_{is} / 2 p_1 = \pi \cdot 13 / 90 = 0.45355 \text{ m}$$

$$X_m = \frac{6 \cdot 4\pi \times 10^{-7} \cdot 2\pi \cdot 50 \cdot (115 \cdot 0.9598)^2 \times 0.45355 \times 1.4}{\pi^2 \times 1.15 \times 0.020 (1 + 0.2) \times 45} = 1.4948 \Omega$$

$$X_{dm} = 1.4948 \times 0.96538 = 1.443 \Omega$$

$$X_{qm} = 1.4948 \times 0.565 = 0.8445 \Omega$$

3. With $X_{1l} = 0.2 \times 1.4948 = 0.2989 \Omega$, the synchronous reactances X_d and X_q are

$$X_d = X_{1l} + X_{dm} = 0.2989 + 1.443 \approx 1.742 \Omega$$

$$X_q = X_{1l} + X_{qm} = 0.2989 + 0.8445 = 1.1434 \Omega$$

4. As the rated efficiency at $\cos \phi_1 = 1$ is $\eta_r = 0.9926$ and using Equation 4.79,

$$\sum p = p_{copper} + p_{Fe} + p_{mec} = S_n \left(\frac{1}{\eta_r} - 1 \right) = 72 \cdot 10^6 \left(\frac{1}{0.9926} - 1 \right) = 536.772 \text{ kW}$$

The stator winding losses p_{copper} are

$$p_{copper} = 3 R_1 I_{1r}^2 = 3 \cdot 0.0125 \cdot 3000^2 = 337.500 \text{ kW}$$

so,

$$p_{Fe} = p_{mec} = \frac{\sum p - p_{copper}}{2} = \frac{536.772 - 337.500}{2} = 99.636 \text{ kW}$$

5. The normalized impedance Z_n is

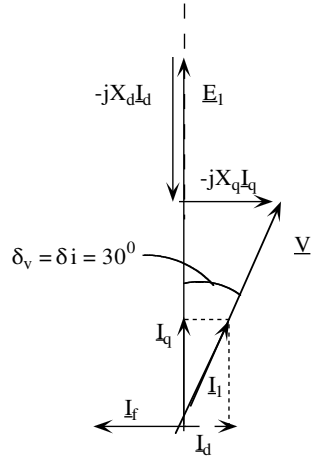
$$Z_n = \frac{V_{1ph}}{I_{1r}} = \frac{13 \cdot 10^3}{\sqrt{3} \cdot 3000} = 2.5048 \Omega$$

$$x_d = \frac{X_d}{Z_n} = \frac{1.742}{2.5048} = 0.695$$

$$x_q = \frac{X_q}{Z_n} = \frac{1.1434}{2.5048} = 0.45648$$

$$r_1 = \frac{R_1}{Z_n} = \frac{0.0125}{2.5048} = 4.99 \times 10^{-3}$$

6. After neglecting all losses, the phasor diagram in [Figure 4.16a](#), for $\cos \phi_1 = 1$, can be shown.



Phasor diagram for $\cos \phi_1 = 1$ and zero losses.

The phasor diagram uses phase quantities in RMS values.

From the adjacent phasor diagram:

$$I_q = \frac{V_1 \sin \delta_v}{X_q} = \frac{13000}{\sqrt{3}} \cdot \frac{0.5}{1.1434} = 3286 \text{ A}$$

$$I_d = -I_q \tan 30^\circ = -3286 \cdot \frac{1}{\sqrt{3}} = -1899.42 \text{ A}$$

$$I_1 = \sqrt{I_d^2 + I_q^2} = 3796 \text{ A!}$$

And, the emf per phase E_1 is

$$E_1 = V_1 \cos \delta_v + X_d |I_d| = \frac{13000}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} + 1.742 \cdot 1899 = 9.808 \text{ kV}$$

$$P_1 = 3V_1 I_1 \cos \phi_1 = 3 \cdot \frac{13000}{\sqrt{3}} \cdot 3796 = 85.372 \text{ MW}$$

$$Q_1 = 3V_1 I_1 \sin \phi_1 = 0$$

It could be inferred that the rated power angle δ_{vr} is smaller than 30° in this practical example.

7. We may use Equation 4.48 to calculate E_1 at no load:

$$E_1 = \frac{\omega_r}{\sqrt{2}} W_1 K_{w1} \Phi_{pole1}$$

$$\Phi_{pole1} = \frac{2}{\pi} B_{g1} \cdot \tau \cdot l_{stack}$$

Then, from Equation 4.20,

$$B_{g1} = B_{gFM} K_{F1}; K_{F1} = \frac{4}{\pi} \sin \frac{\tau_p}{\tau} \frac{\pi}{2}$$

Also, from Equation 4.78,

$$B_{gFM} = \frac{\mu_0 W_F I_F}{K_C g (1 + K_S)}$$

So, gradually,

$$\Phi_{pole1} = \frac{9808 \times \sqrt{2}}{2\pi 50 \times 115 \times 0.9596} = 0.3991 \text{ Wb}$$

$$B_{g1} = \frac{0.3991 \times 3.14}{2 \times 0.45355 \times 1.4} = 0.9868$$

$$B_{gFM} = \frac{0.9868}{\frac{4}{\pi} \sin 0.72 \cdot \frac{\pi}{2}} = 0.8561 \text{ T}$$

$$W_F I_F = \frac{0.8561 \times 1.15 (1 + 0.2) \times 2 \times 10^{-2}}{1.256 \times 10^{-6}} = 18,812 \text{ A turns / pole}$$

Note that the large airgap ($g = 2 \times 10^{-2}$ m) justifies the moderate saturation (iron reluctance) factor $K_S = 0.2$.

The field-winding losses were not considered in the efficiency, as they are covered from a separate power source.

4.9 Autonomous Operation of Synchronous Generators

Autonomous operation of SGs is required by numerous applications. Also, some SG characteristics in autonomous operation, obtained through special tests or by computation, may be used to characterize the SG comprehensively. Typical characteristics at constant speed are as follows:

- No-load saturation curve: $E_1(I_F)$
- Short-circuit saturation curve: $I_{1sc}(I_F)$ for $V_1 = 0$ and $\cos \phi_1 = \text{ct.}$
- Zero-power factor saturation curve: $V_1(I_1); I_F = \text{ct.}$ $\cos \phi_1 = \text{ct.}$

These curves may be computed or obtained from standard tests.

4.9.1 The No-Load Saturation Curve: $E_1(I_F); n = \text{ct.}, I_1 = 0$

At zero-load (stator) current, the excited machine is driven at the speed $n_1 = f_1/p_1$ by a smaller power rating motor. The stator no-load voltage, in fact, the emf (per phase or line) E_1 and the field current are measured. The field current is monotonously raised from zero to a positive value $I_{F\max}$ corresponding to 120 to 150% of rated voltage V_{1r} at rated frequency f_{1r} ($n_{1r} = f_{1r}/p_1$). The experimental arrangement is shown in Figure 4.19a and Figure 4.19b.

At zero-field current, the remanent magnetization of rotor pole iron produces a small emf E_{1r} (2 to 8% of V_{1r}), and the experiments start at point A or A'. The field current is then increased in small increments until the no-load voltage E_1 reaches 120 to 150% of rated voltage (point B, along the trajectory AMB). Then, the field current is decreased steadily to zero in very small steps, and the characteristic evolves along the BNA' trajectory. It may be that the starting point is A', and this is confirmed when I_F

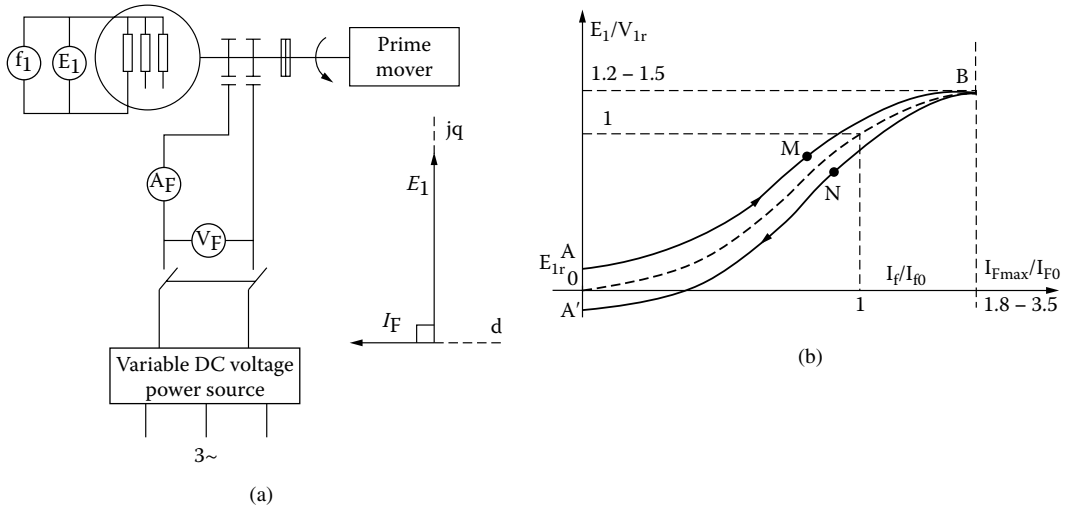


FIGURE 4.19 No-load saturation curve test: (a) the experimental arrangement and (b) the characteristic.

increases from zero, and the emf decreases first and then increases. In this latter case, the characteristic is traveled along the way $A'NBMA$. The hysteresis phenomenon in the stator and rotor cores is the cause of the difference between the rising and falling sides of the curve. The average curve represents the no-load saturation curve.

The increase in emf well above the rated voltage is required to check the required field current for the lowest design power factor at full load (I_{Fmax}/I_{F0}). This ratio is, in general, $I_{Fmax}/I_{F0} = 1.8-3.5$. The lower the lowest power factor at full load and rated voltage, the larger I_{Fmax}/I_{F0} ratio is. This ratio also varies with the airgap-to-pole-pitch ratio (g/τ) and with the number of pole pairs p_1 . It is important to know the corresponding I_{Fmax}/I_{F0} ratio for a proper thermal design of the SG.

The no-load saturation curve may also be computed: either analytically or through finite element method (FEM). As FEM analysis will be dealt with later, here we dwell on the analytical approach. To do so, we draw two typical flux line pairs corresponding to the no-load operation of an SG (Figure 4.20a and Figure 4.20b).

There are two basic analytical approaches of practical interest. Let us call them here the flux-line method and the multiple magnetic circuit method. The simplified flux-line method considers Ampere's law along a basic flux line and applies the flux conservation in the rotor yoke, rotor pole body, and rotor pole shoe, and, respectively, in the stator teeth and yoke.

The magnetic saturation in these regions is considered through a unique (average) flux density and also an average flux line length. It is an approximate method, as the level of magnetic saturation varies tangentially along the rotor-pole body and shoe, in the salient rotor pole, and in the rotor teeth of the nonsalient pole.

The leakage flux lost between the salient rotor pole bodies and their shoes is also approximately considered.

However, if a certain average airgap flux density value B_{gFm} is assigned for start, the rotor pole mmf $W_F I_F$ required to produce it, accounting for magnetic saturation, though approximately, may be computed without any iteration. If the airgap under the rotor salient poles increases from center to pole ends (to produce a more sinusoidal airgap flux density), again, an average value is to be considered to simplify the computation. Once the $B_{gFm}(I_F)$ curve is calculated, the $E_1(I_F)$ curve is straightforward (based on Equation 4.30):

$$E_1(I_F) = \frac{\omega_r}{\sqrt{2}} \times \frac{2}{\pi} \tau B_{gFm}(I_F) K_{F1} \cdot I_{stack} \cdot W_1 K_{W1} [V(RMS)] \quad (4.80)$$

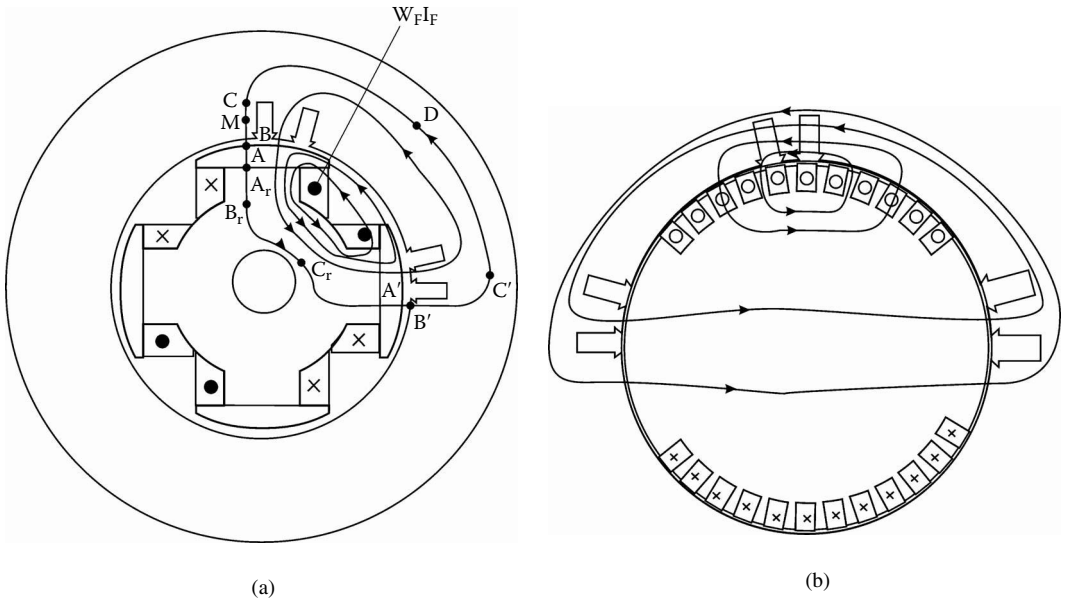


FIGURE 4.20 Flux lines at no load: (a) the salient pole rotor and (b) the nonsalient pole rotor.

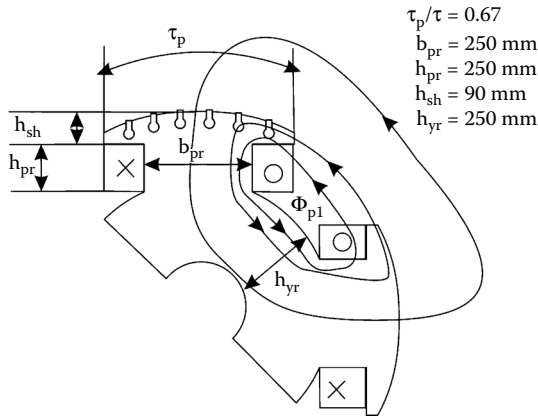


FIGURE 4.21 Rotor geometry and rotor pole leakage flux Φ_{pl} .

The analytical flux-line method is illustrated here through a case study (Example 4.2).

Example 4.2

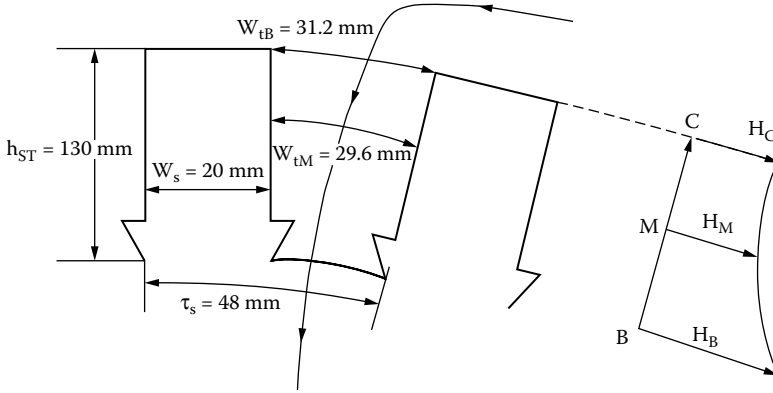
A three-phase salient pole rotor SG with $S_n = 50 \text{ MVA}$, $V_1 = 10,500 \text{ V}$, $n_1 = 428 \text{ rpm}$, and $f_1 = 50 \text{ Hz}$ has the following geometrical data: internal stator diameter $D_r = 3.85 \text{ m}$, $2p_1 = 14$ poles, $l_{stack} \approx 1.39 \text{ m}$, pole pitch $\tau = \pi D_r / 2p_1 = 0.864 \text{ m}$, airgap g (constant) = 0.021 m , $q_1 = \text{six slots/pole/phase}$, open stator slots with $h_s = 0.130 \text{ m}$ (total slot height with 0.006 m reserved for the wedge), $W_s = 0.020 \text{ m}$ (slot width), stator yoke $h_{ys} = 0.24 \text{ m}$, and rotor geometry as in Figure 4.21.

Let us consider only the rated flux density condition, with $B_{gFm1} = 0.850 \text{ T}$. The stator lamination magnetization curve is given in Table 4.1.

Ampere's law along the contour ABCDC'B'A' relates the mmf drop from rotor to rotor pole $F_{AA'}$:

TABLE 4.1 The Magnetization Curve $B(H)$ for the Iron Cores

B(T)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9		
H(A/m)	35	49	65	76	90	106	124	148	177		
B(T)	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
H(A/m)	220	273	356	482	760	1,340	2,460	4,800	8,240	10,200	34,000

**FIGURE 4.22** Stator slot geometry and the no-load magnetic field.

$$F_{AA'} = 2 \left(H_{gFm1} g K_c + \frac{1}{6} (H_B + 4H_M + H_C) h_{st} + H_{YS} l_{YS} \right) \quad (4.81)$$

The airgap magnetic field H_{gFm1} is

$$H_{gFm1} = \frac{B_{gFm1}}{\mu_0} = \frac{0.85}{1.256 \times 10^{-6}} = 0.676 \times 10^6 \text{ A/m} \quad (4.82)$$

The magnetic fields at the stator tooth top, middle, and bottom (H_B , H_M , H_C) are related to Figure 4.22, which shows that the stator tooth is trapezoidal, as the slot is rectangular.

The flux density in the three tooth cross-sections is

$$B_B = B_{gFm1} \cdot \frac{\tau_s}{\tau_s - W_s}; W_s = 0.02 \text{ m}; \tau_s = \frac{\tau}{qm} = \frac{0.8635}{6.3} = 0.048 \text{ m}$$

$$B_M = B_B \cdot \frac{\tau - W_s}{W_{tm}}; W_{tm} = \frac{\pi(D_{is} + h_{st})}{2pqm} - W_s$$

$$B_C = B_B \cdot \frac{(\tau_s - W_s)}{W_{tB}}; W_{tB} = \frac{\pi(D_{is} + 2h_{st})}{2pqm} - W_s$$

Finally,

$$B_B = 0.85 \frac{48}{48.20} = 1.457 \text{ T}$$

$$W_{tm} = \frac{\pi(3.85 + 0.130)}{14 \times 6 \times 3} - 0.02 = 0.0296 \text{ m}$$

$$B_M = 1.457 \cdot \frac{48 - 20}{29.6} = 1.378 \text{ T}$$

$$W_{tB} = \frac{\pi(3.85 + 2 \times 0.130)}{14 \times 6 \times 3} - 0.02 = 0.0312 \text{ m}$$

$$B_C = 1.457 \cdot \frac{(48 - 20)}{31.2} = 1.307 \text{ T}$$

From the magnetization curve (Table 4.1) through linear interpolation, we obtain the following:

- $H_B = 1090.6 \text{ A/m}$
- $H_M = 698.84 \text{ A/m}$
- $H_C = 501.46 \text{ A/m}$

The maximum flux density in stator yoke B_{ys} is

$$B_{ys} = \frac{\tau}{\pi} \cdot \frac{B_{gfm1}}{h_{ys}} = \frac{0.8635}{\pi} \cdot \frac{0.85}{0.24} = 0.974 \text{ T}$$

From Table 4.1, $H_{ys} = 208.82 \text{ A/m}$.

Now, the average length of the flux line in the stator yoke “reduced” to the peak yoke flux density B_{ys} , is approximately

$$l_{ys} \approx \frac{\pi(D_{is} + 2h_{st} + h_{ys})}{4p} \cdot K_{ys}; 0.5 < K_{ys} < 1$$

The value of K_{ys} depends on the level of saturation and other variables. FEM digital simulations may be used to find the value of the “fudge” factor K_{ys} . A reasonable value would be $K_{ys} \approx 2/3$. So,

$$l_{ys} = \frac{\pi(3.85 + 2.013 + 0.24)}{4 \times 7} \times \frac{2}{3} = 0.3252 \text{ m}$$

We may now calculate $F_{AA'}$ from Equation 4.82:

$$\begin{aligned} F_{AA'} &= 2 \left(0.676 \cdot 10^6 \times 2 \cdot 10^{-2} + \frac{1}{6} (1090.6 + 4 \times 698.84 + 501.46) \right) \cdot 0.130 + \\ &+ 208.82 \cdot 0.3252 = 27365.93 \text{ A turns} \end{aligned}$$

Now, the leakage flux Φ_{pl} in the rotor — between rotor poles (Figure 4.21) — is proportional to $F_{AA'}$.

Alternatively, Φ_{pl} may be considered as a fraction of pole flux Φ_p :

$$\Phi_p = \frac{2}{\pi} B_{gFm1} \cdot \tau \cdot l_{stack} = \frac{2}{\pi} 0.85 \cdot 0.8635 \times 0.39 = 0.649 \text{ Wb}$$

$$\Phi_{pl} = K_{sl} \Phi_p; K_{sl} \approx 0.15 - 0.25$$

$$\Phi_{pl} = 0.15 \cdot 0.649 = 0.09747 \text{ T}$$

So, the total flux in the rotor pole Φ_{pr} is

$$\Phi_{pr} = \Phi_p + \Phi_{pl} = 0.649 + 0.09747 = 0.7465 \text{ Wb}$$

The rotor pole shoe is not saturated in the no-load area despite the presence of rotor damper bars, but the pole body and rotor yoke may be saturated. The mmf required to magnetize the rotor F_{rotor} is

$$F_{rotor} = (F_{ABr} + F_{BrCr}) \times 2 = 2(H_{pr} \cdot h_{pr} + H_{yr} \cdot l_{yr}) \quad (4.83)$$

With the rotor pole body width $b_{pr} = 0.25 \text{ m}$, the flux density in the pole body B_{pr} is

$$B_{pr} \approx \frac{\Phi_{pr}}{l_{stack} \cdot b_{pr}} = \frac{0.7465}{1.39 \times 0.25} = 2.148 \text{ T!!}$$

This very large flux density level does not occur along the entire rotor height h_{pr} . At the top of the pole body, approximately, $\Phi_{pr} \approx \Phi_p = 0.649 \text{ Wb}$.

So,

$$(B_{pr})_{Ar} \approx \frac{\Phi_p}{l_{stack} b_{pr}} = \frac{0.649}{1.39 \times 0.25} = 1.8676 \text{ T!!}$$

Consider an average:

$$B_{prav} = \frac{2.148 + 1.8676}{2} = 2 \text{ T!!}$$

For this value, in [Table 4.1](#), we can determine that $H_{pr} = 34,000 \text{ A/m}$.

In the rotor yoke, B_{yr} is

$$B_{yr} = \frac{\Phi_{pr}}{2h_{yr} \cdot l_{stack}} = \frac{0.7465}{2 \times 0.25 \times 1.39} = 1.074 \text{ T!!}$$

So, $H_{yr} = 257 \text{ A/m}$.

The average length of field path in the rotor yoke l_{yr} is

$$I_{yr} \approx \frac{\pi(D_{is} - 2g - 2(h_{sh} + h_{rp}) - h_{yr})}{4p_1} =$$

$$= \frac{\pi(3.85 - 2 \cdot 0.02 - 2(0.09 + 0.25) - 0.25)}{4 \cdot 7} = 0.32185 \text{ m}$$

So, from Equation 4.83, F_{rotor} is

$$F_{rotor} = 2[34,000 \cdot (0.09 + 0.25) + 257 \cdot 0.32185] = 23285 \text{ A turns}$$

Now, the total mmf per two neighboring poles (corresponding to a complete flux line) $2W_F I_F$ is

$$2 \times W_F I_F = F_{AA'} + F_{rotor} = 27,365 + 23,285 = 50,650 \text{ A turns}$$

The airgap mmf requirements are as follows:

$$F_g = 2H_{gFm1} \cdot g = 2 \times 0.676 \times 10^6 + 2 \times 10^{-2} = 27,040 \text{ A turns}$$

The contribution of the iron in the mmf requirement, defined as a saturation factor K_s , is

$$1 + K_s = \frac{2W_F I_F}{F_g} = \frac{50,650}{27,040} = 1.8731$$

So, $K_s = 0.8731$.

For the case in point, the main contribution is placed in the rotor pole. This is natural, as the pole body width b_{pr} must have room in which to place the field windings. So, in general, b_{pr} is around $\tau/3$, at most. The above example illustrates the computational procedure for one point of the no-load magnetization curve $B_{gFm1}(I_F)$. Other points may be calculated in a similar way. A more precise solution, at the price of larger computation time, may be obtained through the multiple magnetic circuit method [4], but real precision results require FEM, as shown in [Chapter 5](#).

4.9.2 The Short-Circuit Saturation Curve $I_1 = f(I_f)$; $V_1 = 0$, $n_1 = n_r = \text{ct}$.

The short-circuit saturation curve is obtained by driving the excited SG at rated speed n_r with short-circuited stator terminals ([Figure 4.23a](#) through [Figure 4.23d](#)). The field DC I_F is varied downward gradually, and both I_F and stator current I_{sc} are measured. In general, measurements for 100%, 75%, 50%, and 25% of rated current are necessary to reduce the winding temperature during that test. The results are plotted in [Figure 4.23b](#).

From the voltage equation 4.54 with $V_1 = 0$ and $I_1 = I_{3sc}$, one obtains the following:

$$\underline{E}_1(I_f) = R_1 I_{3sc} + jX_d I_{dsc} + jX_q I_{qsc} \quad (4.84)$$

$$\underline{E}_1(I_f) = -jX_F I_F$$

Neglecting stator resistance and observing that, with zero losses, $I_{3sc} = I_{dsc}$, as $I_{qsc} = 0$ (zero torque), we obtain

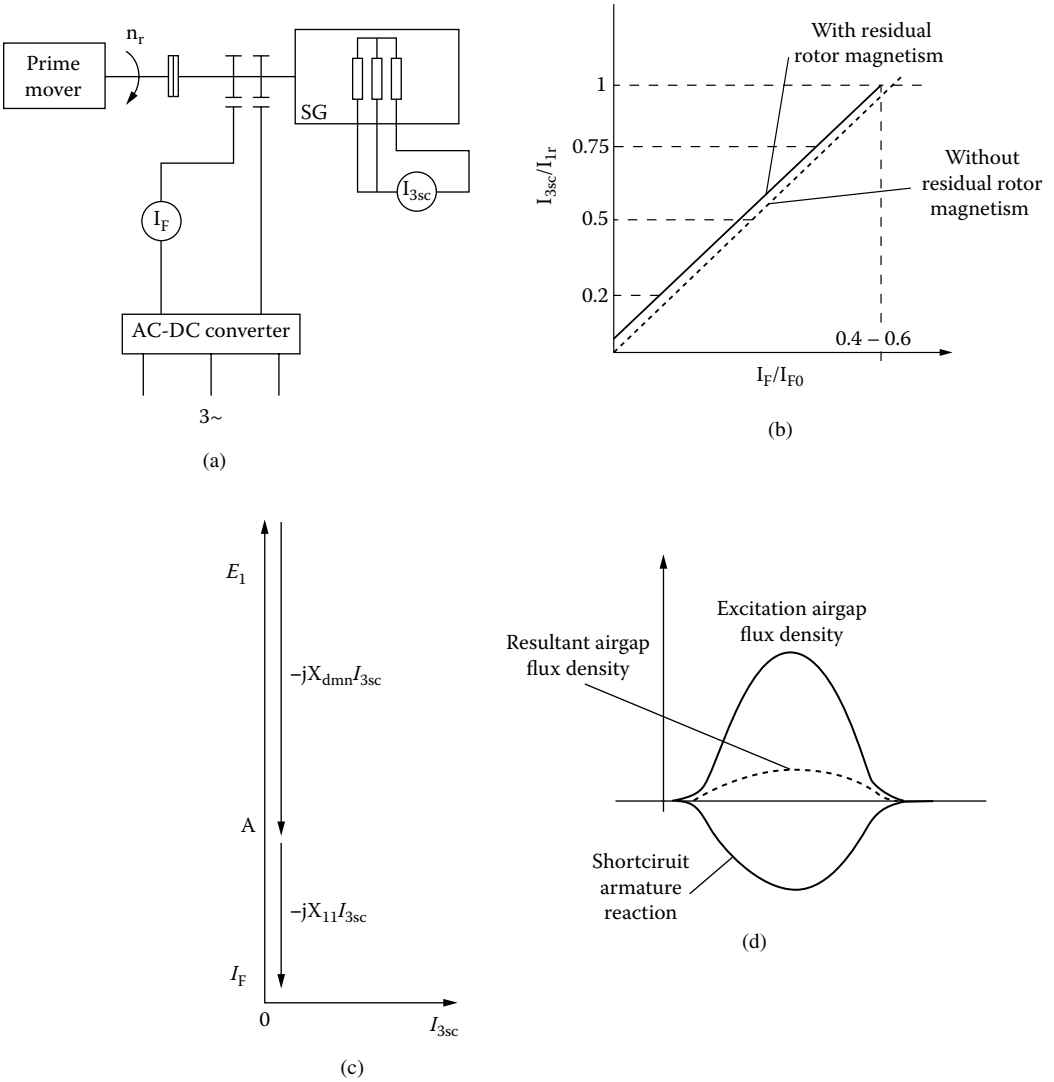


FIGURE 4.23 The short-circuit saturation curve: (a) experimental arrangement, (b) the characteristics, (c) phasor diagram with $R_1 = 0$, and (d) airgap flux density (slotting neglected).

$$\underline{E}_1(I_F) = jX_d I_{3sc} \quad (4.85)$$

The magnetic circuit is characterized by very low flux density (Figure 4.23d). This is so because the armature reaction strongly reduces the resultant emf E_{1res} to

$$\underline{E}_{1res} = -jX_{l1} \cdot \underline{I}_{3sc} = E_1 - jX_{dm} I_{3sc} \quad (4.86)$$

which represents a low value on the no-load saturation curve, corresponding to an equivalent small field current (Figure 4.24):

$$I_{F0} = I_F - I_{3sc} \cdot \frac{X_{dm}}{X_{FA}} = OA - AC \quad (4.87)$$

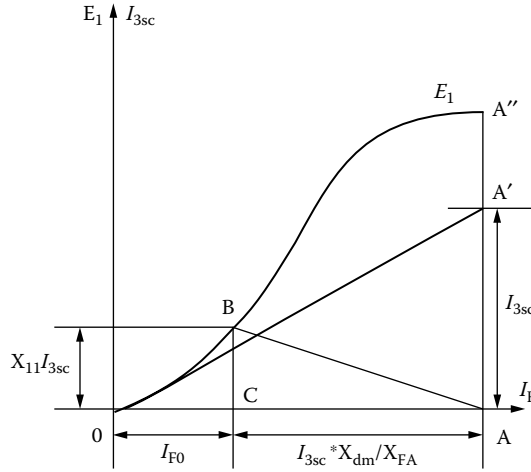


FIGURE 4.24 The short-circuit triangle.

Adding the no-load saturation curve, the short-circuit triangle may be portrayed (Figure 4.24). Its sides are all quasi-proportional to the short-circuit current.

By making use of the no-load and short-circuit saturation curves, saturated values of d axis synchronous reactance may be obtained:

$$X_{ds} = \frac{E_1(I_F)}{I_{3sc}(I_F)} = \frac{\overline{AA''}}{\overline{AA'}} \quad (4.88)$$

Under load, the magnetization state differs from that of the no-load situation, and the value of X_{ds} from Equation 4.88 is of limited practical utilization.

4.9.3 Zero-Power Factor Saturation Curve $V_1(I_F)$; $I_1 = \text{ct.}$, $\cos\phi_1 = 0$, $n_1 = n_r$

Under zero power factor and zero losses, the voltage Equation 4.54 becomes

$$V_1 = E_1 - jX_d I_d; I_1 = I_d; I_q = 0 \quad (4.89)$$

Again, for pure reactive load and zero losses, the electromagnetic torque is zero; and so is I_q .

An underexcited synchronous machine acting as a motor on no load is generally used to represent the reactive load for the SG under zero power factor operation with constant stator current I_d .

The field current of the SG is reduced simultaneously with the increase in field current of the underexcited no-load synchronous motors (SM), to keep the stator current I_d constant (at rated value), while the terminal voltage decreases. In this way, $V_1(I_F)$ for constant I_d is obtained (Figure 4.25a and Figure 4.25b).

The abscissa of the short-circuit triangle OCA is moved at the level of rated voltage, then a parallel OB' to OB is drawn that intersects the no-load curve at B' . The vertical segment OB' is defined as follows:

$$X_p I_1 = \overline{B'C} \quad (4.90)$$

Though we started with the short-circuit triangle in our geometrical construction, $BC < B'C$ because magnetic saturation conditions are different. So, in fact, $X_p > X_{11}$, in general, especially in salient pole rotor SGs.

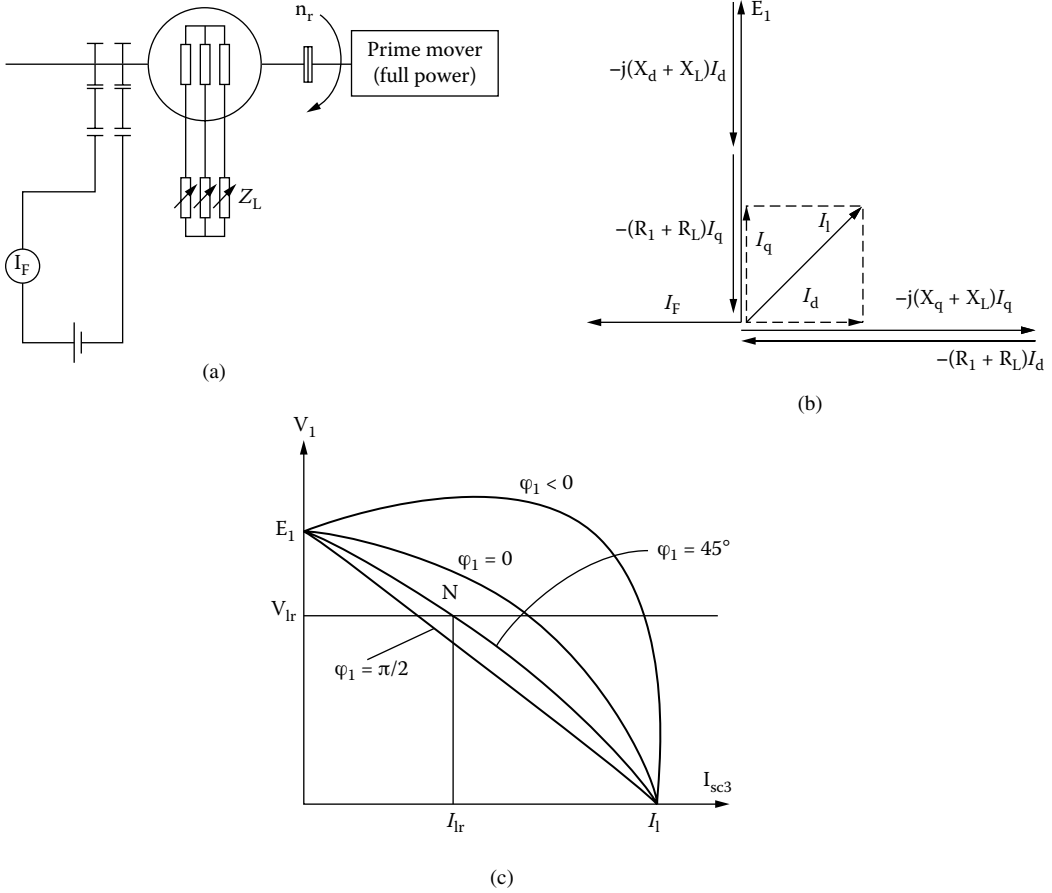


FIGURE 4.26 $V_1 - I_1$ curve: (a) the experimental arrangement, (b) the phasor diagram for load Z_L , and (c) the curves.

with I_F given, E_1 is extracted from the no-load saturation curve. Then, with $\cos \phi_1$ given, we may choose to modify R_L (load resistance) only as X_L is

$$X_L = R_L \tan \phi_1 \quad (4.93)$$

Then, Equation 4.92 can be simply solved to calculate I_d and I_q . The phase current I_1 is

$$I_1 = \sqrt{I_d^2 + I_q^2} \quad (4.94)$$

Finally, the corresponding terminal voltage V_1 is

$$V_1 = \frac{R_L}{\cos \phi_1} \cdot I_1 \quad (4.95)$$

Typical $V_1(I_1)$ curves are shown in Figure 4.26c. The voltage decreases with load (I_1) for resistive ($\phi_1 = 0$) and resistive-inductive ($\phi_1 > 0$) load, and it increases and then decreases for resistive-capacitive load ($\phi_1 < 0$). Such characteristics may be used to calculate the voltage regulation ΔV_1 :

$$(\Delta V_1)_{I_1/\cos\phi_1/I_F} = \frac{E_1 - V_1}{E_1} = \frac{\text{no_load_voltage} - \text{load_voltage}}{\text{no_load_voltage}} \quad (4.96)$$

Autonomous SGs are designed to provide operation at rated load current and rated voltage and a minimum (lagging) power factor $\cos\phi_{1\min} = 0.6\text{--}0.8$ (point N on Figure 4.26c). It should be evident that I_{1r} should be notably smaller than I_{3sc} .

Consequently,

$$\frac{X_{dsat}}{Z_n} = x_{dsat} < 1; Z_n = \frac{V_N}{I_N} \quad (4.97)$$

The airgap in SGs for autonomous operation has to be large to secure such a condition. Consequently, notable field current mmf is required. Thus, the power loss in the field winding increases. This is one reason to consider permanent magnet rotor SGs for autonomous operation for low to medium power units, even though full-power electronics are needed.

Note that for calculations with errors (below 1 to 2%) when using Equation 4.92, careful consideration of the magnetic saturation level that depends simultaneously on I_F , I_d , I_q must be considered. This subject will be treated in more detail in Chapter 5.

4.10 Synchronous Generator Operation at Power Grid (in Parallel)

SGs in parallel constitute the basis of a regional, national, or continental electric power system (grid). SGs have to be connected to the power grid one by one.

For the time being, we will suppose that the power grid is of infinite power, that is, of fixed voltage, frequency, and phase. In order to connect the SGs to the power grid without large current and power transients, the amplitude, frequency, sequence, and phase of the SG no-load voltages have to coincide with the same parameters of the power grid. As the power switch does not react instantaneously, some transients will always occur. However, they have to be limited. Automatic synchronization of the SG to the power grid is today performed through coordinated speed (frequency and phase) and field current control (Figure 4.27).

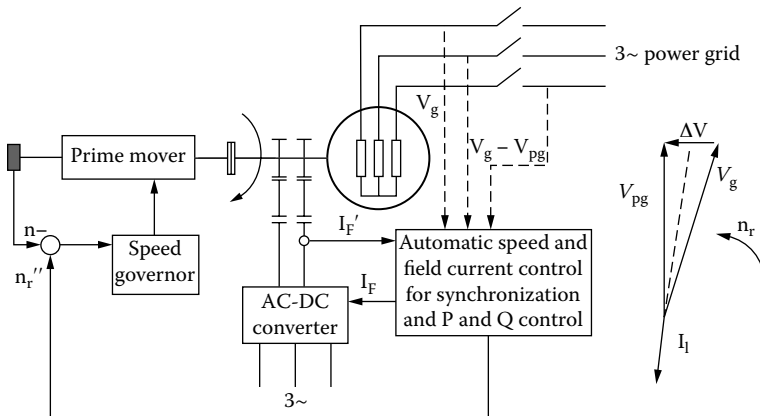


FIGURE 4.27 Synchronous generator connection to the power grid.

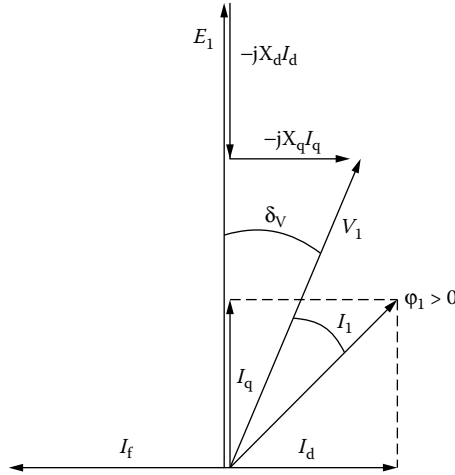


FIGURE 4.28 Synchronous generator phasor diagram (zero losses).

The active power transients during connection to the power grid may be positive (generating) or negative (motoring) (Figure 4.27).

4.10.1 The Power/Angle Characteristic: $P_e(\delta_v)$

The power (internal) angle δ_v is the angle between the terminal voltage V_1 and the field-current-produced emf E_1 . This angle may be calculated for the autonomous and for the power-grid-connected generator. Traditionally, the power/angle characteristic is calculated and widely used for power-grid-connected generators, mainly because of stability computation opportunities. For a large power grid, the voltage phasors in the phasor diagram are fixed in amplitude and phase. For clarity, we neglect the losses in the SG. We repeat here the phasor diagram in Figure 4.17a but with $R_1 = 0$ (Figure 4.28).

The active and reactive powers P_1 , Q_1 from Equation 4.60 and Equation 4.61 with $R_1 = 0$ become

$$P_1 = 3E_1 I_q + 3(X_{dm} - X_q) I_d I_q \quad (4.98)$$

$$Q_1 = -3E_1 I_d - 3X_d I_d^2 - 3X_q I_q^2 \quad (4.99)$$

From Figure 4.28,

$$I_d = \frac{V_1 \cos \delta_v - E_1}{X_d}; I_q = \frac{V_1 \sin \delta_v}{X_q} \quad (4.100)$$

With Equation 4.100, Equation 4.98 and Equation 4.99 become the following:

$$P_1 = \frac{3E_1 V_1 \sin \delta_v}{X_d} + \frac{3}{2} V_1^2 \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta_v \quad (4.101)$$

$$Q_1 = \frac{3E_1 V_1 \cos \delta_v}{X_d} - 3V_1^2 \left(\frac{\cos^2 \delta_v}{X_d} + \frac{\sin^2 \delta_v}{X_q} \right)$$

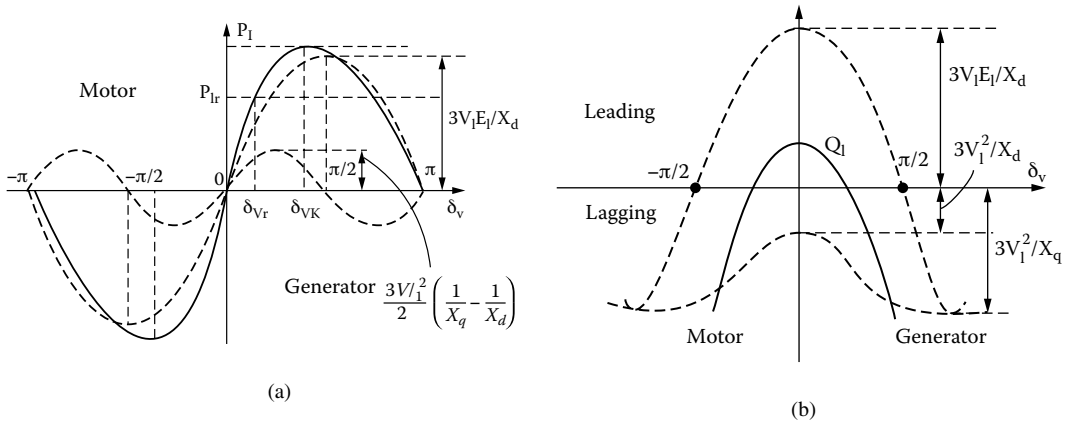


FIGURE 4.29 (a) Active P_1 and (b) reactive Q_1 powers vs. power angle δ_v .

The unity power factor is obtained with $Q_1 = 0$, that is,

$$(E_1)_{Q_1=0} = V_1 \left(\cos \delta_v + \frac{X_d}{X_q} \frac{\sin^2 \delta_v}{\cos \delta_v} \right) \quad (4.102)$$

For the same power angle δ_v and V_1 , E_1 should be larger for the salient pole rotor SG, as $X_d > X_q$. The active power has two components: one due to the interaction of stator and rotor fields, and the second one due to the rotor magnetic saliency ($X_d > X_q$).

As in standard salient pole rotor SGs, $X_d/X_q < 1.7$, the second term in P_e , called here saliency active power, is relatively small unless the SG is severely underexcited: $E_1 \ll V_1$. For given E_1 , V_1 , the SG reactive and active power delivery depend on the power (internal) angle δ_v (Figure 4.29a and Figure 4.29b).

The graphs in Figure 4.29a and Figure 4.29b warrant the following remarks:

- The generating and motoring modes are characterized (for zero losses) by positive and, respectively, negative power angles.
- As δ_v increases up to the critical value δ_{vK} , which corresponds to maximum active power delivery P_{1K} , the reactive power goes from leading to lagging for given emf E_1 , V_1 frequency (speed) ω_1 .
- The reactive power is independent of the sign of the power angle δ_v .
- In salient pole rotor SGs, the maximum power P_{1K} for given V_1 , E_1 and speed, is obtained for a power angle $\delta_{vK} < 90^\circ$, while for nonsalient pole rotor SGs, $X_d = (1 - 1.05)X_q$, $\delta_{vK} \approx 90^\circ$.
- The rated power angle δ_{vr} is chosen around 22 to 30° for nonsalient pole rotor SGs and around 30 to 40° for salient pole rotor SGs. The lower speed, higher relative inertia, and stronger damper cage of the latter might secure better stability, which justifies the lower power reserve (or ratio P_{1K}/P_{1r}).

4.10.2 The V-Shaped Curves: $I_1(I_F)$, $P_1 = \text{ct.}$, $V_1 = \text{ct.}$, $n = \text{ct.}$

The V-shaped curves represent a family of $I_1(I_F)$ curves, drawn at constant V_1 , speed (ω_1), with active power P_1 as a parameter. The computation of a V-shaped curve is straightforward once $E_1(I_F)$ — the no-load saturation curve — and X_d and X_q are known. Unfortunately, when I_F varies from low to large values, so does I_1 (that is, I_d , I_q); magnetic saturation varies, despite the fact that, basically, the total flux linkage $\Psi_s \approx V_1/\omega_1$ remains constant. This is due to rotor magnetic saliency ($X_d \neq X_q$), where local saturation conditions vary notably. However, to a first approximation, for constant V_1 and ω_1 (that is

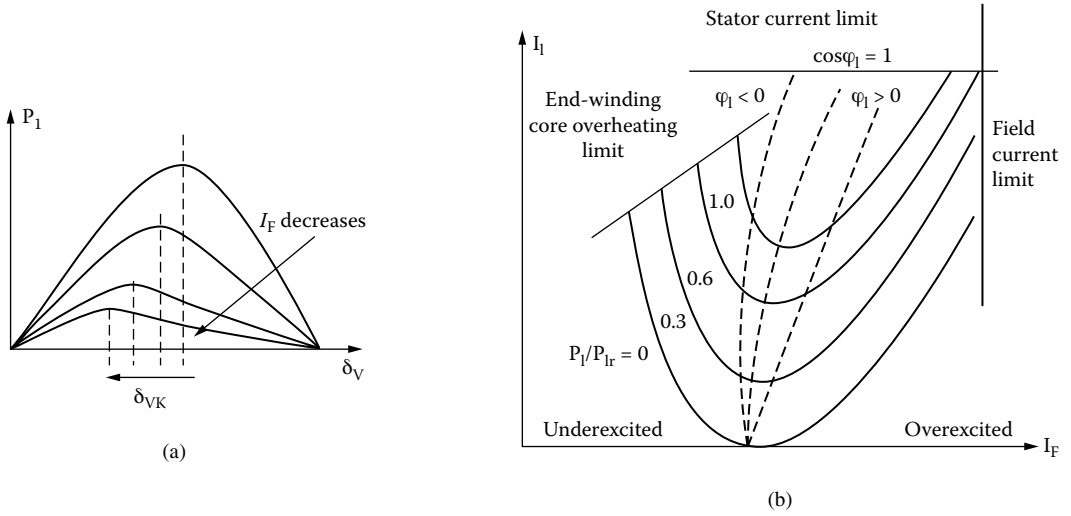


FIGURE 4.30 V-shaped curves: (a) P_1/δ_v assisting curves with I_F as parameter and (b) the $I_1(I_F)$ curves for constant P_1 .

Ψ_1), with E_1 calculated at a first fixed total flux, the value of M_{FA} stays constant, and thus, $E_1 \approx M_{FA} \cdot I_F$

$\cdot \omega_r \approx C_{FA} \cdot I_F$; $C_{FA} \approx \frac{V_{1r}}{I_{F0}}$. I_{F0} is the field current value that produces $E_1 = V_{1r}$ at no load.

For given I_F , $E_1 = C_{FA} \times I_F$ and P_1 assigned a value from (4.101), we may compute δ_v . Then, from Equation 4.103, the corresponding stator current I_1 can be found:

$$I_1 = \sqrt{I_d^2 + I_q^2} = \sqrt{\left(\frac{-E_1 + V_1 \cos \delta_v}{X_d} \right)^2 + \left(\frac{V_1 \sin \delta_v}{X_q} \right)^2} \quad (4.103)$$

As expected, for given active delivered power, the minimum value of stator current is obtained for a field current I_F corresponding to unity power factor ($Q_1 = 0$). That is, $(E_1)_{I_{1\min}} = (E_1)_{Q_1=0}$ may be determined from Equation 4.103 with δ_v already known from Equation 4.101. Then, $I_{RK} = E_1/C_{FA}$. The maximum power angle admitted for a given power P_1 limits the lowest field current admissible for steady state.

Finally, graphs as shown in Figure 4.30a and Figure 4.30b are obtained.

Knowing the field current lower limit, for given active power, is paramount in avoiding an increase in the power angle above δ_{VK} . In fact, δ_{VK} decreases with an increase in P_1 .

4.10.3 The Reactive Power Capability Curves

The maximum limitation of I_F is due to thermal reasons. However, the SG heating depends on both I_1 and I_F , as both winding losses are very important. Also, I_1 , I_F , and δ_v determine the core losses in the machine at a given speed.

When a reactive power request is increased, the increase in I_F raises the field-winding losses and thus the stator-winding losses (the active power P_1) have to be limited.

The rationale for V-shaped curves may be continued to find the reactive power Q_1 for the given P_1 and I_F . As shown in Figure 4.30, there are three distinct thermal limits: I_F limit (vertical), I_1 limit (horizontal), and the end-winding overheating (inclined) limit at low values of field current. To explain this latest, rather obscure, limitation, refer to Figure 4.31.

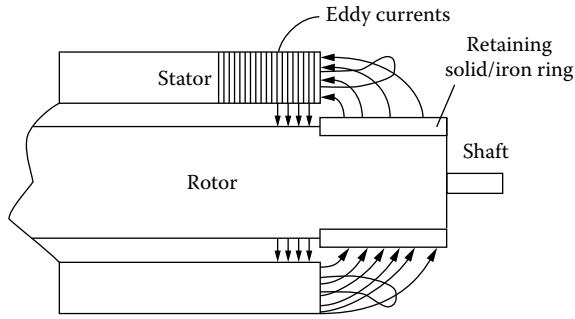


FIGURE 4.31 End-region field path for the underexcited synchronous generator.

For the underexcited SG, the field-current- and armature-current-produced fields have angles smaller than 90° (the angle between I_F and I_A in the phasor diagram). Consequently, their end-winding fields more or less add to each other. This resultant end-region field enters at 90° the end-region stator laminations and produces severe eddy current losses that thermally limit the SG reactive power absorption ($Q_1 < 0$). This phenomenon is so strong because the retaining ring solid iron eddy currents (produced solely by the stator end-windings currents) are small and thus incapable of attenuating severely the end-region resultant field. This is because the solid iron retaining ring is not saturated magnetically, as the field current is small. When the SG is overexcited, this phenomenon is not important, because the stator and rotor fields are opposite (I_F and I_A phase-angle shift is above 90°) and the retaining magnetic ring is saturated by the large field current. Consequently, the stator end-windings-current-produced field in the stator penetrates deeply into the retaining rings, producing large eddy currents that further attenuate this resultant field in the end-region zone (the known short-circuit transformer effect on inductance). The $Q_1(P_1)$ curves are shown in Figure 4.32.

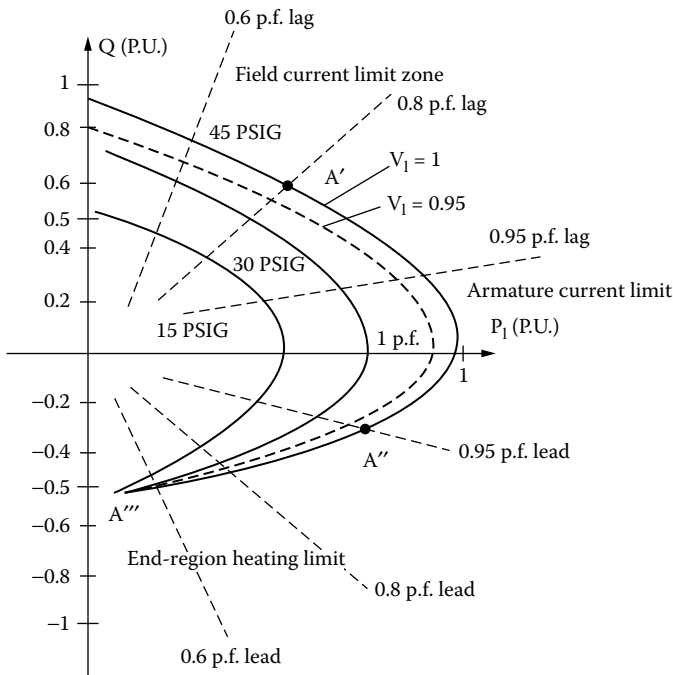


FIGURE 4.32 Reactive power capability curves for a hydrogen-cooled synchronous generator.

The reduction of hydrogen pressure leads to a reduction of reactive and active power capability of the machine.

As expected, the machine reactive power absorption capability ($Q_1 < 0$) is notably smaller than reactive power capability. Both the end-region lamination loss limitation and the rise of the power angle closer to its maximum limitation, seem to be responsible for such asymmetric behavior (Figure 4.32).

4.10.4 Defining Static and Dynamic Stability of Synchronous Generators

The fact that SGs require constant speed to deliver electric power at constant frequency introduces special restrictions and precautionary measures to preserve SG stability, when tied to an electric power system (grid). The problem of stability is complex. To preserve and extend it, active speed and voltage (active and reactive power) closed-loop controls are provided. We will deal in some detail with stability and control in Chapter 6. Here, we introduce the problem in a more phenomenological manner. Two main concepts are standard in defining stability: static stability and dynamic stability.

The static stability is the property of an SG to remain in synchronism to the power grid in the presence of slow variations in the shaft power (output active power, when losses are neglected). According to the rising side of the $P_1(\delta_v)$ curve (Figure 4.28), when the mechanical (shaft) power increases, so does the power angle δ_v , as the rotor slowly advances the phase of E_1 , with the phase of $\underline{V}_1$ fixed. When δ_v increases, the active power delivered electrically, by the SG, increases.

In this way, the energy balance is kept, and no important energy increment is accumulated in the inertia of the SG. The speed remains constant, but when P_1 increases, so does δ_v . The SG is statically stable if $\partial P_1 / \partial \delta_v > 0$.

We denote by P_{1s} this power derivative with angle and call it synchronization power:

$$P_{1s} = \frac{\partial P_1}{\partial \delta_v} = 3E_1 V_1 \cos \delta_v - 3V_1^2 \left(\frac{1}{X_d} - \frac{1}{X_q} \right) \cos 2\delta_v \quad (4.104)$$

P_{1s} is maximum at $\delta_v = 0$ and decreases to zero when δ_v increases toward δ_{vK} , where $P_{1s} = 0$.

At the extent that the field current decreases, so does δ_{vK} , and thus, the static stability region diminishes. In reality, the SG is allowed to operate at values of δ_v , notably below δ_{vK} , to preserve dynamic stability.

The dynamic stability is the property of the SG to remain in synchronism (with the power grid) in the presence of quick variations of shaft power or of electric load short-circuit. As the combined inertia of SGs and their prime movers is relatively large, the speed and power angle transients are much slower than electrical (current and voltage) transients. So, for example, we can still consider the SG under electromagnetic steady state when the shaft power (water admission in a hydraulic turbine) varies to produce slow-speed and power-angle transients. The electromagnetic torque T_e is thus, approximately,

$$T_e \approx \frac{P_1 \cdot p_1}{\omega_r} = \frac{3P_1}{\omega_r} \left[\frac{E_1 V_1 \sin \delta_v}{X_d} + \frac{V_1^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta_v \right] \quad (4.105)$$

Consider a step variation of shaft power from P_{sh1} to P_{sh2} (Figure 4.33a and Figure 4.33b) in a lossless SG.

The SG power angle should vary slowly from δ_{v1} to δ_{v2} . In reality, the power angle δ_v will overshoot δ_{v2} and, after a few attenuated oscillations, will settle at δ_{v2} if the machine remains in synchronism.

Neglect the rotor damper cage effects that occur during transients. The motion equation is then written as follows:

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_{shaft} - T_e; \omega_r - \omega_{r0} = \frac{d\delta_v}{dt} \quad (4.106)$$

with ω_{r0} equal to the synchronous speed.

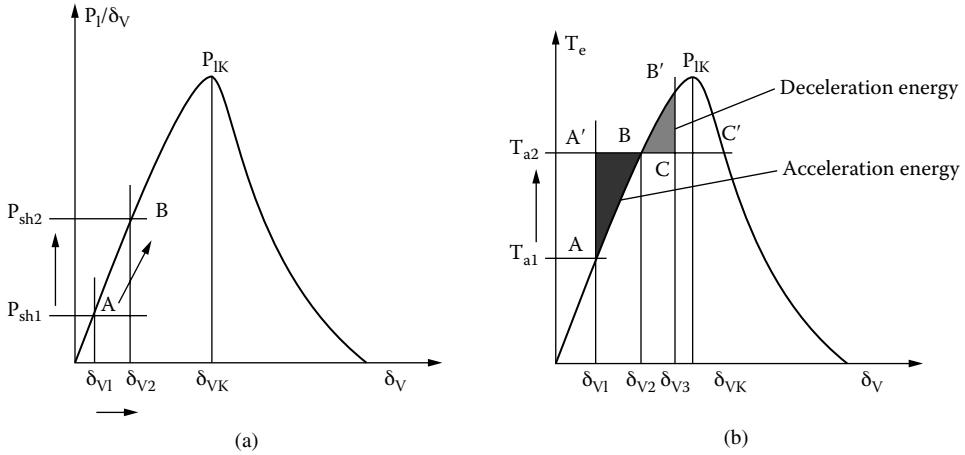


FIGURE 4.33 Dynamic stability: (a) $P_1(\delta_V)$ and (b) the area criterion.

By multiplying Equation 4.106 by $d\delta_V/dt$, one obtains

$$d\left(\frac{J}{2P_1}\left(\frac{d\delta_V}{dt}\right)^2\right) = (T_{shaft} - T_e)d\delta_V = \Delta T \cdot d\delta_V = dW \quad (4.107)$$

Equation 4.107 illustrates the variation of kinetic energy of the prime-mover generator set translated in an acceleration area AA'B and a deceleration area BB'C:

$$W_{AB} = \text{area_of_} AA'B \text{_triangle} = \int_{\delta_{V1}}^{\delta_{V2}} (T_{shaft} - T_e)d\delta_V \quad (4.108)$$

$$W_{AB'} = \text{area_of_} BB'C \text{_triangle} = \int_{\delta_{V2}}^{\delta_{V3}} (T_{shaft} - T_e)d\delta_V \quad (4.109)$$

Only when the two areas are equal to each other is there hope that the SG will come back from B' to B after a few attenuated oscillations. Attenuation comes from the asynchronous torque of damper cage currents, neglected so far. This is the so-called criterion of areas.

The maximum shaft torque or electric power step variation that can be accepted with the machine still in synchronism is shown in Figure 4.34a and Figure 4.34b and corresponds to the case when point C coincides with C'.

Let us illustrate the dynamic stability with the situation in which there is a loaded SG at power angle δ_{V1} . A three-phase short-circuit occurs at δ_{V1} , with its transients attenuated very quickly such that the electromagnetic torque is zero ($V_1 = 0$, zero losses also). So, the SG starts accelerating until the short-circuit is cleared at δ_{Vsc} , which corresponds to a few tens of a second at most. Then, the electromagnetic torque T_e becomes larger than the shaft torque, and the SG decelerates. Only if

$$\text{Area_of_} \overline{ABCD} \geq \text{Area_of_} CB'B'' \quad (4.110)$$

are there chances for the SG to remain in synchronism, that is, to be dynamically stable.

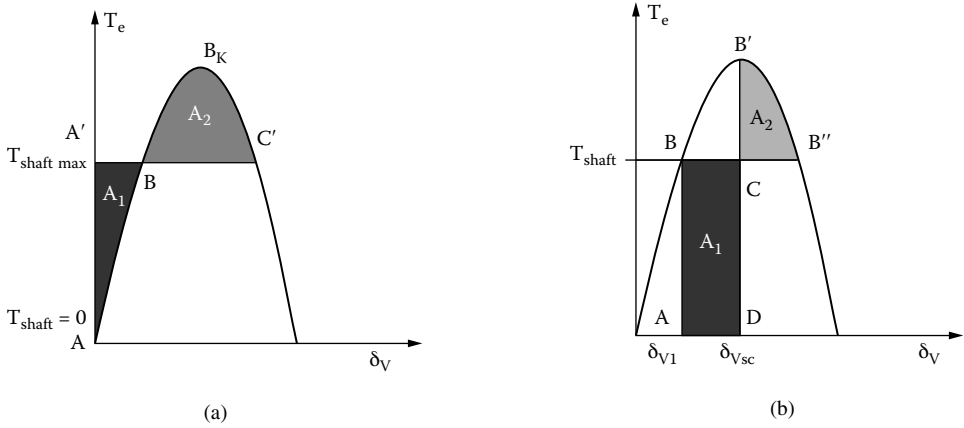


FIGURE 4.34 Dynamic stability ideal limits: (a) maximum shaft torque step variation from zero and (b) maximum short-circuit clearing time (angle: $\delta_{Vsc} - \delta_{V1}$) from load.

4.11 Unbalanced-Load Steady-State Operation

SGs connected to the power grid, but especially those in autonomous applications, often operate on unbalanced three-phase loads. That is, the stator currents in the three phases have different amplitudes, and their phasing differs from 120° :

$$\begin{aligned}
 I_A(t) &= I_1 \cos(\omega_1 t - \gamma_1) \\
 I_B(t) &= I_2 \cos\left(\omega_1 t - \frac{2\pi}{3} - \gamma_2\right) \\
 I_C(t) &= I_3 \cos\left(\omega_1 t + \frac{2\pi}{3} - \gamma_3\right)
 \end{aligned} \tag{4.111}$$

For balanced load, $I_1 = I_2 = I_3$ and $\gamma_1 = \gamma_2 = \gamma_3$. These phase currents may be decomposed in direct, inverse, and homopolar sets according to Fortesque's transform (Figure 4.35).

$$\begin{aligned}
 \underline{I}_{A+} &= \frac{1}{3}(\underline{I}_A + a\underline{I}_B + a^2\underline{I}_C); a = e^{j\frac{2\pi}{3}} \\
 \underline{I}_{A-} &= \frac{1}{3}(\underline{I}_A + a^2\underline{I}_B + a\underline{I}_C); a^2 = e^{-j\frac{2\pi}{3}} \\
 \underline{I}_{A0} &= \frac{1}{3}(\underline{I}_A + \underline{I}_B + \underline{I}_C) \\
 \underline{I}_{B+} &= a^2 \underline{I}_{A+}; \\
 \underline{I}_{C+} &= a \underline{I}_{A+}; \\
 \underline{I}_{B-} &= a \underline{I}_{A-}; \\
 \underline{I}_{C-} &= a^2 \underline{I}_{A-}
 \end{aligned} \tag{4.112}$$

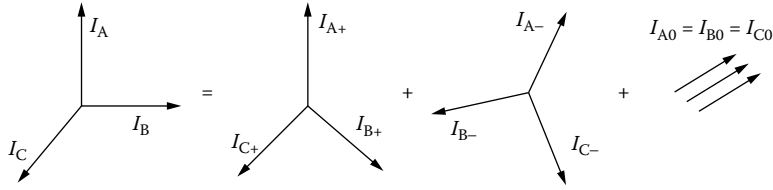


FIGURE 4.35 The symmetrical component sets.

Unfortunately, the superposition of the flux linkages of the current sets is admissible only in the absence of magnetic saturation. Suppose that the SG is nonsaturated and lossless ($R_1 = 0$). For the direct components $\underline{I}_{A1}$, $\underline{I}_{B1}$, $\underline{I}_{C1}$, which produce a forward-traveling mmf at rotor speed, the theory unfolded so far still holds. So, we will write the voltage equation for phase A and direct component of current $\underline{I}_{A1}$:

$$\underline{V}_{A+} = \underline{E}_{A+} - jX_d \underline{I}_{dA+} - jX_q \underline{I}_{qA+} \quad (4.113)$$

The inverse (negative) components of stator currents $\underline{I}_A^-$, $\underline{I}_B^-$, and $\underline{I}_C^-$ produce an mmf that travels at opposite rotor speed $-\omega_r$.

The relative angular speed of the inverse mmf with respect to rotor speed is thus $2\omega_r$. Consequently, voltages and currents are induced in the rotor damper windings and in the field winding at $2\omega_r$ frequency, in general. The behavior is similar to an induction machine at slip $S = 2$, but which has nonsymmetrical windings on the rotor and nonuniform airgap. We may approximate the SG behavior with respect to the inverse component as follows:

$$\begin{aligned} \underline{I}_A \cdot \underline{Z}_- + \underline{U}_{A-} &= \underline{E}_{A-} \\ \underline{Z}_- &= R_- + jX_- \end{aligned} \quad (4.114)$$

Unless the stator windings are not symmetric or some of the field coils have short-circuited turns $\underline{E}_{A-} = 0$, $\underline{Z}_-$ is the inverse impedance of the machine and represents a kind of multiple winding rotor induction machine impedance at $2\omega_r$ frequency.

The homopolar components of currents produce mmfs in the three phases that are phase shifted spatially by 120° and have the same amplitude and time phasing. They produce a zero-traveling field in the airgap and thus do not interact with the rotor in terms of the fundamental component. The corresponding homopolar impedance is $Z_0 \approx R_1 + jX_0$, and

$$X_0 < X_{ll} \quad (4.115)$$

So,

$$X_d > X_q > X_- > X_{ll} > X_0 \quad (4.116)$$

The stator equation for the homopolar set is as follows:

$$j\underline{I}_{A0} X_0 + \underline{V}_{A0} = 0 \quad (4.117)$$

Finally,

$$\underline{V}_A = \underline{V}_{A+} + \underline{V}_{A-} + \underline{V}_{A0} \quad (4.118)$$

Similar equations are valid for the other two phases. We assimilated here the homopolar with the stator leakage reactance ($X_{1l} \approx X_0$). The truth is that this assertion is not valid if chorded coils are used, when $X_0 < X_{1l}$. It seems that, due to the placement of stator winding in slots, the stator homopolar mmf has a steplike distribution with $\tau/3$ as half-period and does not rotate; it is an AC field. This third-space harmonic-like mmf may be decomposed in a forward and backward wave and move both with respect to rotor and induce eddy currents, at least in the damper cage. Additional losses occur in the rotor. As we are not prepared by now theoretically to calculate Z_- and X_0 , we refer to some experiments to measure them so that we get some confidence in using the above theory of symmetrical components.

4.12 Measuring X_d , X_q , Z_- , Z_0

We will treat here some basic measurement procedures for SG reactances: X_d , X_q , Z_- , Z_0 . For example, to measure X_d and X_q , the open-field-winding SG, supplied with symmetric forward voltages (ω_{r0} , frequency) through a variable-ratio transformer, is driven at speed ω_r , which is very close to but different from the stator frequency ω_{r0} (Figure 4.36a and Figure 4.36b):

$$\omega_r = \omega_{r0} \cdot (1.01 - 1.02) \quad (4.119)$$

We need not precisely measure this speed, but notice the slow pulsation in the stator current with frequency $\omega_r - \omega_{r0} \approx (0.01 - 0.02)\omega_{r0}$.

Identifying the maxima and minima in the stator voltage $V_A(t)$ and current $I_A(t)$ leads to approximate values of X_d and X_q :

$$X_d \approx \frac{V_{A \max}}{I_{A \min}}; X_q = \frac{V_{A \min}}{I_{A \max}} \quad (4.120)$$

The slip $S = (\omega_r - \omega_{r0})/\omega_{r0}$ has to be very small so that the currents induced in the rotor damper cage may be neglected. If they are not negligible, X_d and X_q are smaller than in reality due to the damper eddy current screening effect.

The saturation level will be medium if currents around or above the rated value are used.

Identifying the voltage and current maxima, even if the voltage and current are digitally acquired and are off-line processed in a computer, is doable with practical precision.

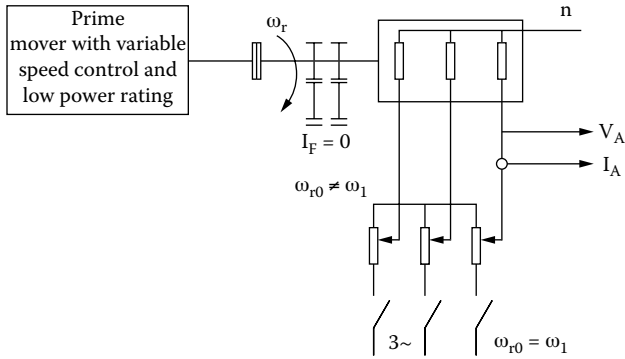
The inverse (negative) sequence impedance Z_- may be measured by driving the rotor, with the field winding short-circuited, at synchronous speed ω_r , while feeding the stator with a purely negative sequence of low-level voltages (Figure 4.37). The power analyzer is used to produce the following:

$$|Z_-| = \frac{V_{A-}}{I_{A-}}; R_- = \frac{(P_-)_{\text{phase}}}{I_{A-}^2} \quad (4.121)$$

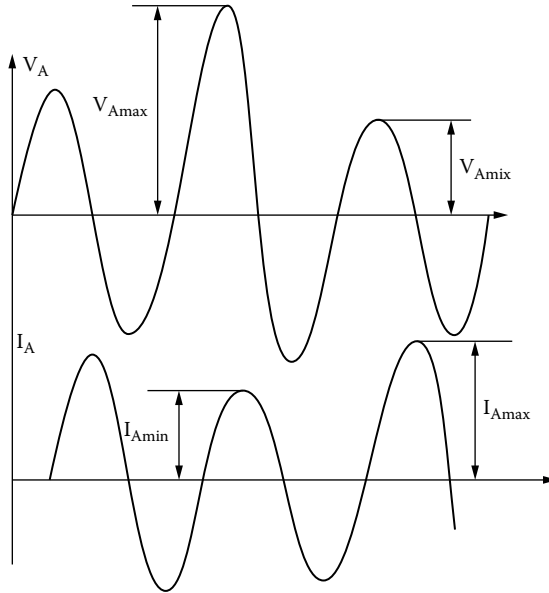
$$X_- = \sqrt{|Z_-|^2 - (R_-)^2} \quad (4.122)$$

Again, the frequency of currents induced in the rotor damper and field windings is $2\omega_{r0} = 2\omega_1$, and the corresponding slip is $S = 2.0$. Alternatively, it is possible to AC supply the stator between two phases only:

$$\underline{Z_-} \approx \frac{U_{AB}}{2I_{A-}} \quad (4.123)$$



(a)



(b)

FIGURE 4.36 Measuring X_d and X_q : (a) the experimental arrangement and (b) the voltage and current waveforms.

This time the torque is zero and thus the SG stays at standstill, but the frequency of currents in the rotor is only $\omega_{r0} = \omega_1$. (The negative sequence impedance will be addressed in detail in [Chapter 8](#) of *Synchronous Generators* on SG testing.) The homopolar impedance $\underline{Z}_0$ may be measured by supplying the stator phases connected in series from a single-phase AC source. The test may be made at zero speed or at rated speed ω_{r0} ([Figure 4.38](#)). For the rated speed test, the SG has to be driven at shaft. The power analyzer yields the following:

$$|\underline{Z}_0| = \frac{3V_{A0}}{3I_{A0}}; R_0 = \frac{P_0}{3I_{A0}^2}; X_0 = \sqrt{|\underline{Z}_0|^2 - R_0^2} \quad (4.124)$$

A good portion of R_0 is the stator resistance R_1 so $R_0 \approx R_1$.

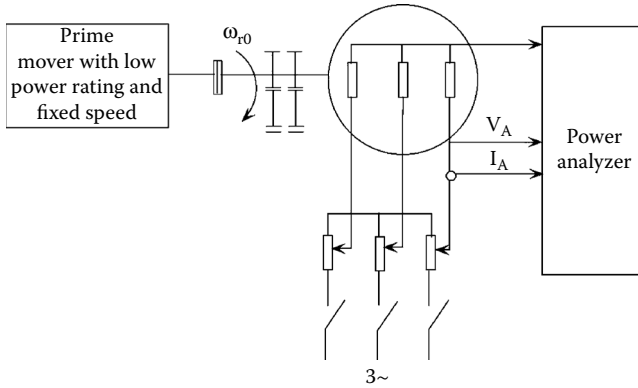


FIGURE 4.37 Negative sequence testing for $\underline{Z}_-$.

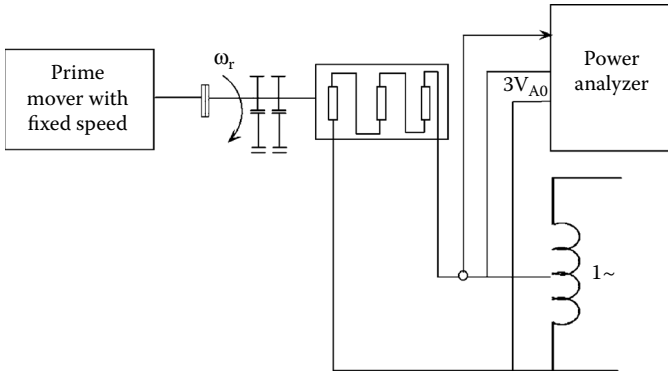


FIGURE 4.38 Measuring homopolar $\underline{Z}_0$.

The voltage in measurements for $\underline{Z}$ and $\underline{Z}_0$ should be made low to avoid high currents.

4.13 The Phase-to-Phase Short-Circuit

The three-phase (balanced) short-circuit was already investigated in a previous paragraph with the current I_{3sc} :

$$I_{3sc} = \frac{E_1}{X_d} \quad (4.125)$$

The phase-to-phase short-circuit is a severe case of unbalanced load. When a short-circuit between two phases occurs, with the third phase open, the currents are related to each other as follows (Figure 4.39a):

$$I_B = -I_C = I_{sc2}; V_B = V_C; I_A = 0 \quad (4.126)$$

From Equation 4.109, the symmetrical components of $\underline{I}_A$ are as follows:

$$\underline{I}_{A+} = \frac{1}{2}(a\underline{I}_B + a^2\underline{I}_C) = \frac{1}{3}(a - a^2)\underline{I}_{sc2} = \frac{+j}{\sqrt{3}}\underline{I}_{sc2} \quad (4.127)$$

$$\underline{I}_{A-} = -\underline{I}_{A+}; \underline{I}_{A0} = 0$$

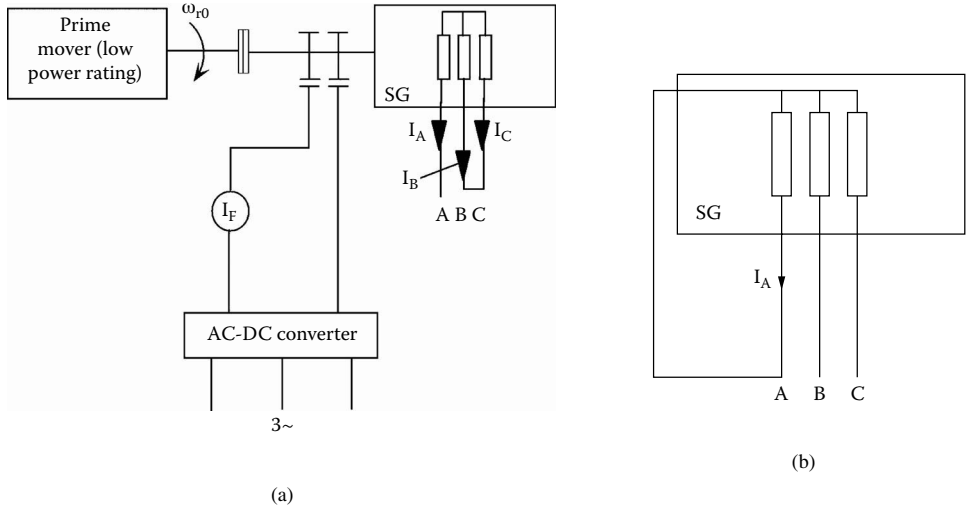


FIGURE 4.39 Unbalanced short-circuit: (a) phase-to-phase and (b) single-phase.

The star connection leads to the absence of zero-sequence current.

The terminal voltage of phase A, V_A , for a nonsalient pole machine ($X_d = X_q = X_+$) is obtained from Equation 4.115 with Equation 4.112 and Equation 4.113:

$$\begin{aligned} \underline{V}_A &= \underline{V}_{A+} + \underline{V}_{A-} + \underline{V}_{A0} = \underline{E}_{A+} - jX_+ \cdot \underline{I}_{A+} - \underline{Z}_- \underline{I}_{A-} = \\ &= \underline{E}_{A+} - \frac{j}{\sqrt{3}} \underline{I}_{sc2} (jX_+ - \underline{Z}_-) \end{aligned} \quad (4.128)$$

In a similar way,

$$\begin{aligned} \underline{V}_B &= a^2 \underline{V}_{A+} + a \underline{V}_{A-} = a^2 \underline{E}_{A+} - \frac{j \underline{I}_{sc2}}{\sqrt{3}} (ja^2 X_+ - a \underline{Z}_-) \\ \underline{V}_C &= a \underline{V}_{A+} + a^2 \underline{V}_{A-} = a \underline{E}_{A+} - \frac{j \underline{I}_{sc2}}{\sqrt{3}} (aj X_+ - a^2 \underline{Z}_-) \end{aligned} \quad (4.129)$$

But, $\underline{V}_B = \underline{V}_C$ and thus

$$\underline{E}_{A+} = -\frac{j \underline{I}_{sc2}}{\sqrt{3}} (jX_+ + \underline{Z}_-) \quad (4.130)$$

and

$$\underline{V}_A = \frac{2j}{\sqrt{3}} \underline{I}_{sc2} \underline{Z}_- = -2 \underline{V}_B \quad (4.131)$$

Finally,

$$jX_+ + \underline{Z}_- = -j\sqrt{3} \frac{\underline{E}_{A+} (I_F)}{\underline{I}_{sc2}} \quad (4.132)$$

A few remarks are in order:

- Equation 4.132, with the known no-load magnetization curve, and the measured short-circuit current I_{sc2} , apparently allows for the computation of negative impedance if the positive one $jX_+ = jX_d$, for nonsalient pole rotor SG, is given. Unfortunately, the phase shift between $\underline{E}_{A1}$ and $\underline{I}_{sc2}$ is hard to measure. Thus, if we only:

$$\underline{Z}_- = -jX_- \quad (4.133)$$

Equation 4.132 becomes usable as

$$X_+ + X_- = \frac{E_{A+} \cdot \sqrt{3}}{I_{sc2}} \quad (4.134)$$

RMS values enter Equation 4.134.

- Apparently, Equation 4.131 provides a good way to compute the negative impedance $\underline{Z}_-$ directly, with $\underline{V}_A$ and $\underline{I}_{sc2}$ measured. Their phase shift can be measured if the SG null point is used as common point for $\underline{V}_A$ and $\underline{I}_{sc2}$ measurements.
- During a short-circuit, even in phase to phase, the airgap magnetic flux density is small and distorted. So, it is not easy to verify (Equation 4.131), unless the voltage V_A and current I_{sc2} are first filtered to extract the fundamental.
- Only Equation 4.132 is directly usable to approximate X_- , with X_+ unsaturated known. As $X_+ \gg X_-$ for strong damper cage rotors, the precision of computing X_- from the sum $(X_+ + X_-)$ is not so good
- In a similar way, as above for the single-phase short-circuit (Figure 4.39b),

$$X_+ + X_- + X_0 \approx \frac{3E_{A+}(I_F)}{I_{sc1}} \quad (4.135)$$

with $X_+ > X_- > X_0$.

- To a first approximation,

$$I_{sc3} : I_{sc2} : I_{sc1} \approx 3 : \sqrt{3} : 1 \quad (4.136)$$

for an SG with a strong damper cage rotor.

- E_{A+} should be calculated for the real field current I_F , but, as during short-circuit the real distortion level is low, the unsaturated value of X_{FA} should be used: $E_{A+} = I_f(X_{FA})_{\text{unsaturated}}$.
- Small autonomous SGs may have the null available for single-phase loads; thus, the homopolar component shows up.
- The negative sequence currents in the stator produce double-frequency-induced currents in the rotor damper cage and in the field windings. If the field winding is supplied from a static power converter, the latter prevents the occurrence of AC currents in the field winding. Consequently, notable overvoltages may occur in the latter. They should be considered when designing the field-winding power electronics supply. Also, the double-frequency currents in the damper cage, produced by the negative component set, have to be limited, as they affect the rotor overtemperature. So, the ratio I/I_+ , that is the level of current unbalance, is limited by standards below 10 to 12%.
- A similar phenomenon occurs in autonomous SGs, where the acceptable level of current unbalance I/I_f is given as a specification item and then considered in the thermal design. Finally, experiments are needed to make sure that the SG can really stand the predicted current unbalance.

- The phase-to-phase or single-phase short-circuits are extreme cases of unbalanced load. The symmetrical components method presented here can be used for actual load unbalance situations where the $+$, $-$, 0 current components sets may be calculated first. A numerical example follows.

Example 4.3

A three-phase lossless two-pole SG with $S_n = 100$ KVA, at $V_{ll} = 440$ V and $f_l = 50$ Hz, has the following parameters: $x_+ = x_d = x_q = 0.6$ P.U., $x_- = 0.2$ P.U., $x_0 = 0.12$ P.U. and supplies a single-phase resistive load at rated current. Calculate the load resistance and the phase voltages V_A , V_B , V_C if the no-load voltage $E_{ll} = 500$ V.

Solution

We start with the computation of symmetrical current components sets (with $I_B = I_C = 0$):

$$I_{A+} = I_{A-} = I_{A0} = I_r / 3$$

The rated current for star connection I_r is

$$I_r = \frac{S_n}{\sqrt{3}V_{ll}} = \frac{100000}{\sqrt{3} \cdot 440} \approx 131 \text{ A}$$

The nominal impedance Z_n is

$$Z_n = \frac{U_{ll}}{\sqrt{3}I_r} = \frac{440}{\sqrt{3} \cdot 131} = 1.936 \text{ } \Omega$$

So,

$$X_+ = Z_n x_+ = 1.936 \times 0.6 = 1.1616 \text{ } \Omega$$

$$X_- = Z_n x_- = 1.936 \times 0.2 = 0.5872 \text{ } \Omega$$

$$X_0 = Z_n x_0 = 1.936 \times 0.12 = 0.23232 \text{ } \Omega$$

From Equation 4.112, the positive sequence voltage equation is as follows:

$$\underline{V}_{A+} = \underline{E}_{A+} - jX_+ \underline{I}_r / 3$$

From Equation 4.113,

$$\underline{V}_{A-} = -jX_- \underline{I}_r / 3$$

Also, from Equation 4.116,

$$\underline{V}_{A0} = -jX_0 \underline{I}_r / 3$$

The phase of voltage $\underline{V}_A$ is the summation of its components:

$$\underline{V}_A = \underline{E}_{A+} - j(X_+ + X_- + X_0) \underline{I}_r / 3$$

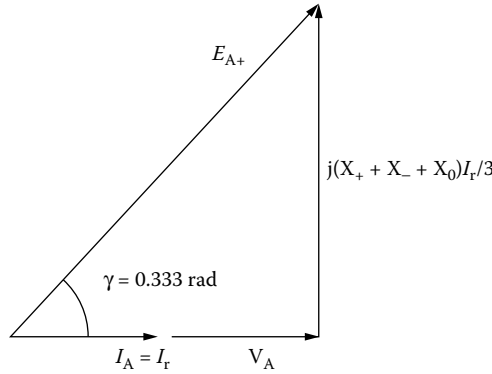


FIGURE 4.40 Phase A phasor diagram.

As the single-phase load was declared as resistive, I_r is in phase with $\underline{V}_A$, and thus,

$$\underline{E}_{A+} = \underline{V}_{A+} + j(X_+ + X_- + X_0) \frac{I_r}{3}$$

A phasor diagram could be built as shown in Figure 4.40.

With $E_{A+} = 500/\sqrt{3} = 288.675$ V and $I_r = 131$ A known, we may calculate the phase voltage of loaded phase, V_A :

$$\begin{aligned} V_A &= \sqrt{E_{A+}^2 - \left[(X_+ + X_- + X_0) I_r / 3 \right]^2} = \sqrt{(288.675)^2 - \left[(1.1616 + 0.3872 + 0.23232) \cdot 131 / 3 \right]^2} \\ &= 278.348 \text{ V} \end{aligned}$$

The voltages along phases B and C are

$$\begin{aligned} \underline{V}_B &= \underline{E}_{A+} e^{-j\frac{2\pi}{3}} - jX_+ \frac{I_r}{3} e^{-j\frac{2\pi}{3}} - jX_- \frac{I_r}{3} e^{+j\frac{2\pi}{3}} - jX_0 \frac{I_r}{3} = \\ \underline{V}_C &= \underline{E}_{A+} e^{+j\frac{2\pi}{3}} - jX_+ \frac{I_r}{3} e^{+j\frac{2\pi}{3}} - jX_- \frac{I_r}{3} e^{-j\frac{2\pi}{3}} - jX_0 \frac{I_r}{3} = \\ \underline{E}_{A+} &= E_{A+} \cdot e^{j\gamma_0}; \gamma_0 = 0.333 \text{ rad} \end{aligned}$$

The real axis falls along $\underline{V}_A$ and $\underline{I}_A$, in the horizontal direction:

$$\begin{aligned} \underline{V}_B &= -83.87 - j \times 270 \text{ [V]} \\ \underline{V}_C &= -188.65 + j213.85 \text{ [V]} \end{aligned}$$

The phase voltages are no longer symmetric ($V_A = 278$ V, $V_B = 282.67$ V, $V_C = 285$ V). The voltage regulation is not very large, as $x_+ = 0.6$, and the phase voltage unbalance is not large either, because the homopolar reactance is usually small, $x_0 = 0.12$. Also small is X_- due to a strong damper cage on the rotor. A small x_+ presupposes a notably large airgap; thus, the field-winding mmf should be large enough to produce acceptable values of flux density in the airgap on no load ($B_{gFm} = 0.7\text{--}0.75$ T) in order to secure a reasonable volume SG.

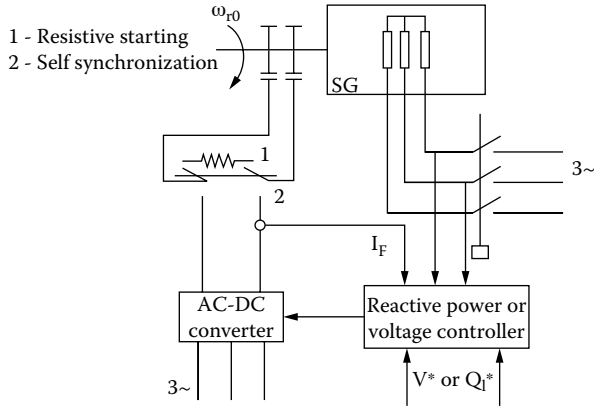


FIGURE 4.41 Synchronous condenser.

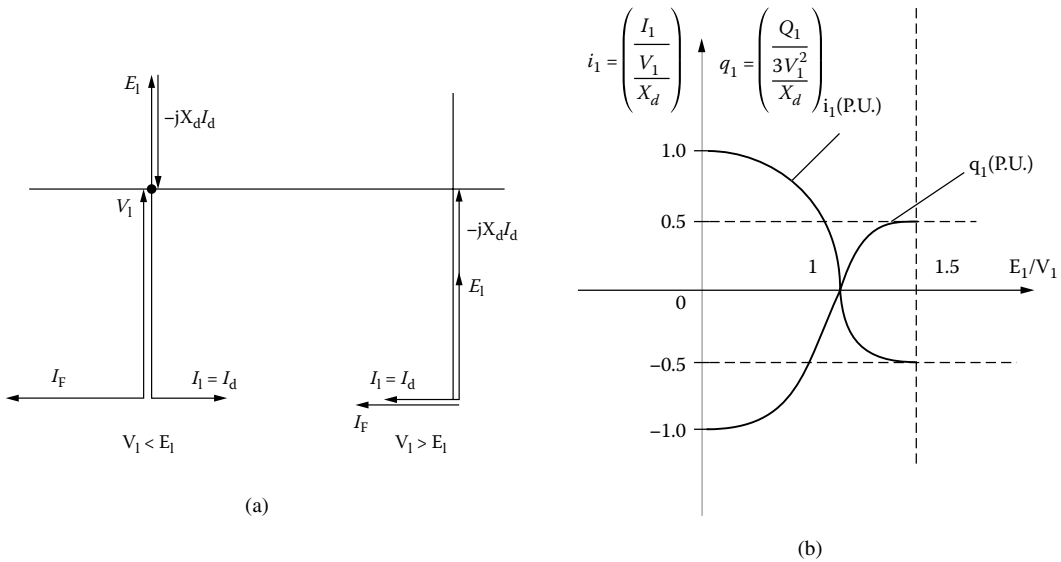


FIGURE 4.42 (a) Phasor diagrams and (b) reactive power of synchronous condenser.

4.14 The Synchronous Condenser

As already pointed out, the reactive power capability of a synchronous machine is basically the same for motor or generating mode (Figure 4.28b). It is thus feasible to use a synchronous machine as a motor without any mechanical load, connected to the local power grid (system), to “deliver” or “drain” reactive power and thus contribute to overall power factor correction or (and) local voltage control. The reactive power flow is controlled through field current control (Figure 4.41).

The phasor diagram (with zero losses) springs from voltage Equation 4.54 with $I_q = 0$ and $R_1 = 0$ (Figure 4.42a and Figure 4.42b):

$$\underline{V}_1 = \underline{E}_1 - jX_d I_d; \underline{I}_1 = \underline{I}_d \quad (4.137)$$

The reactive power Q_1 (Equation 4.102), with $\delta_v = 0$, is

$$Q_1 = 3V_1 \frac{(E_1 - V_1)}{X_d} = -3V_1 I_d \quad (4.138)$$

$$Q_1 = \frac{3V_1^2}{X}; X = \frac{X_d}{E_1 / V_1 - 1} \quad (4.139)$$

As expected, Q_1 changes sign at $E_1 = V_1$ and so does the current:

$$I_d = (V_1 - E_1) / X_d; X = \begin{cases} > 0 & \text{for } E_1 / V_1 > 1 \\ < 0 & \text{for } E_1 / V_1 < 1 \end{cases} \quad (4.140)$$

Negative I_d means demagnetizing I_d or $E_1 > V_1$. As magnetic saturation depends on the resultant magnetic field, for constant voltage V_1 , the saturation level stays about the same, irrespective of field current I_F . So,

$$E_1 \approx \omega_r (M_{FA})_{V_1} \cdot I_F \quad (4.141)$$

Also, X_d should not vary notably for constant voltage V_1 . The maximum delivered reactive power depends on I_d , but the thermal design should account for both stator and rotor field-winding losses, together with core losses located in the stator core.

It seems that the synchronous condenser should be designed at maximum delivered (positive) reactive power $Q_{1\max}$:

$$Q_{1\max} = 3 \frac{V_1}{X_d} [E_{1\max}(I_{F\max}) - V_1]; I_1 = \frac{E_{1\max} - V_1}{X_d} \quad (4.142)$$

To reduce the size of such a machine acting as a no-load motor, two pole rotor configurations seem to be appropriate. The synchronous condenser is, in fact, a positive/negative reactance with continuous control through field current via a low power rating AC–DC converter. It does not introduce significant voltage or current harmonics in the power systems. However, it makes noise, has a sizeable volume, and needs maintenance. These are a few reasons for the increase in use of pulse-width modulator (PWM) converter controlled capacitors in parallel with inductors to control voltage in power systems. Existing synchronous motors are also used whenever possible, to control reactive power and voltage locally while driving their loads.

4.15 Summary

- Large and medium power SGs are built with DC excitation windings on the rotor with either salient or nonsalient poles.
- Salient rotor poles are built for $2p_1 > 4$ poles and nonsalient rotor poles for $2p_1 = 2, 4$.
- The stator core of SGs is made of silicon-steel laminations (generally 0.5 mm thick), with uniform slotting. The slots house the three-phase windings.
- The stator core is made of one piece only if the outer diameter is below 1 m; otherwise, it is made of segments. Sectionable cores are wound section by section, and the wound sections are mounted together at the user's site.
- The slots in SGs are generally open and provided with nonmagnetic or magnetic wedges (to reduce emf harmonic content).

- Stator windings are of single- and double-layer types and are made of lap (multiturn) coils or the bar-wave (single-turn) coils (to reduce the lengthy connections between coils).
- Stator windings are generally built with integer slots/pole/phase q ; only for a large number of poles $2p_1 > 16$ to 20, q may be fractionary: 3.5, 4.5 (to reduce the emf harmonics content).
- The symmetric AC currents of stator windings produce a positive mmf wave that travels with the ω_1/p_1 angular speed (with respect to the stator) $\omega_1 = 2\pi f_1$, with f_1 equal to the frequency of currents.
- The core of salient pole rotors is made of a solid iron pole wheel spider on top of which $2p_1$ salient poles usually made of laminations (1 mm thick in general) are placed. The poles are attached to the pole wheel spider through hammer or dove-tail keybars or bolts and screws with end plates.
- Nonsalient pole rotors are made of solid iron with machined radial slots over two thirds of periphery to house distributed field-winding coils. Constrained costs and higher peripheral speeds have led to solid cores for nonsalient poles rotors with $2p_1 = 2, 4$ poles.
- The rotor poles are provided with additional (smaller) slots filled with copper or brass bars short-circuited by partial or total end rings. This is the damper winding.
- The airgap flux density produced by the rotor field windings has a fundamental and space harmonics. They are to be limited in order to reduce the stator emf (no load voltage) harmonics. The larger airgap under the salient poles is used for the scope. Uniform airgap is used for nonsalient poles, because their distributed field coils produce lower harmonics in the airgap flux density. The design airgap flux density flat top value is about 0.7 to 0.8 T in large and medium power SGs. The emf harmonics may be further reduced by the type of stator winding (larger or fractionary q , chorded coils).
- The airgap flux density of the rotor field winding currents is a traveling wave at rotor speed $\Omega_r = \omega_r/p_1$.
- When $\omega_r = \omega_1$, the stator AC current and rotor DC current airgap fields are at standstill with each other. These conditions lead to an interaction between the two fields, with nonzero average electromagnetic torque. This is the speed of synchronism or the synchronous speed.
- When an SG is driven at speed ω_r (electrical rotor angular speed; $\Omega_r = \omega_r/p_1$ is the mechanical rotor speed), the field rotor DC currents produce emfs in the stator windings with frequency ω_1 that is $\omega_1 = \omega_r$. If a balanced three-phase load is connected to the stator terminals, the occurring stator currents will naturally have the same frequency $\omega_1 = \omega_r$; their mmf will, consequently, produce an airgap traveling field at the speed $\omega_1 = \omega_r$. Their phase shift with respect to phase emfs depends on load character (inductive-resistive or capacitive-resistive) and on SG reactances (not yet discussed). This is the principle of the SG.
- The airgap field of stator AC currents is called the armature reaction.
- The phase stator currents may be decomposed in two components (I_d, I_q), one in phase with the emf and the other at 90° . Thus, two mmfs are obtained, with airgap fields that are at standstill with respect to the moving rotor. One along the d (rotor pole) axis, called longitudinal, and the other one along the q axis, called transverse. This decomposition is the core of the two-reaction theory of SGs.
- The two stator mmf fields are tied to rotor d and q axes; thus, their cyclic magnetization reactances X_{dm} and X_{qm} may be easily calculated. Leakage reactances are added to get X_d and X_q , the synchronous reactances. With zero damper currents and DC field currents on the rotor, the steady-state voltage equation is straightforward:

$$\underline{I}_1 R_1 + V_1 = \underline{E}_1 - jX_d \underline{I}_d - jX_q \underline{I}_q; \underline{I}_1 = \underline{I}_d + \underline{I}_q$$

- The SG “delivers” both active and reactive power, P_1 and Q_1 . They both depend on X_d, X_q , and R_1 and on the power angle δ_v , which is the phase angle between the emf and the terminal voltage (phase variables).

- Core losses may be included in the SG equations at steady state as pure resistive short-circuited stator fictitious windings with currents that are produced by the resultant airgap or stator phase linkage.
- The SG loss components are stator winding losses, stator core losses, rotor field-winding losses, additional losses (mainly in the rotor damper cage), and mechanical losses. The efficiency of large SGs is very good (above 98%, total, including field-winding losses).
- The SGs may operate in stand-alone applications or may be connected to the local (or regional) power system. No-load, short-circuit, zero-power factor saturation curves, together with the output $V_1(I_1)$ curve, fully characterize stand-alone operation with balanced load. Voltage regulation tends to be large with SGs as the synchronous reactances in P.U. (x_d or x_q) are larger than 0.5 to 0.6, to limit the rotor field-winding losses.
- Operation of SGs at the power system is characterized by the angular curve $P_1(\delta_v)$, V-shaped curves $I_1(I_F)$ for $P_1 = \text{ct.}$, and the reactive power capability $Q_1(P_1)$.
- The $P_1(\delta_v)$ curve shows a single maximum value at $\delta_{vK} \leq 90^\circ$; this critical angle decreases when the field current I_F decreases for constant stator terminal voltage V_1 and speed.
- Static stability is defined as the property of SG to remain at synchronism for slow shaft torque variations. Basically, up to $\delta_v = \delta_{vK}$, the SG is statically stable.
- The dynamic stability is defined as the property of the SG to remain in synchronism for fast shaft torque or electric power (short-circuiting until clearing) transients. The area criterion is used to forecast the reserve of dynamic stability for each transient. Dynamic stability limits the rated power angle 22 to 40° , much less than its maximum value $\delta_{vK} \leq 90^\circ$.
- The stand-alone SG may encounter unbalanced loads. The symmetrical components (Fortesque) method may be applied to describe SG operation under such conditions, provided the saturation level does not change (or is absent). Impedances for the negative and zero components of stator currents, $\underline{Z}_-$ and $\underline{Z}_0$, are defined, and basic methods to measure them are described. In general, $|\underline{Z}_+| > |\underline{Z}_-| > |\underline{Z}_0|$, and thus, the stator phase voltage imbalance under unbalanced loads is not very large. However, the negative sequence stator currents induce voltages and produce currents of double stator frequency in the rotor damper cage and field winding. Additional losses are present. They have to be limited to keep rotor temperature within reasonable limits. The maximum I_-/I_+ ratio is standardized (for power system SGs) or specified (for stand-alone SGs).
- The synchronous machine acting as a motor with no shaft load is used for reactive power absorption (I_F small) or delivery (I_F large). This regime is called a synchronous condenser, as the machine is seen by the local power system either as a capacitor (I_F large, overexcited $E_1 > V_1$) or as an inductor (I_F small, underexcited machine $E_1/V_1 < 1$). Its role is to raise or control the local power factor or voltage in the power system.

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